Ring Accelerator Physics for Non Accelerator Specialists

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Outline

1. Light source properties vs electron beam performance
2. Stochasticity of photoemission
3. Distribution of circulating electron beam
4. Approach to coherent X-rays

Towards survival

Even any member in an experiment-related division or BL scientists, who are not the accelerator specialists, should understand the relation between the SR properties and electron beam performances,

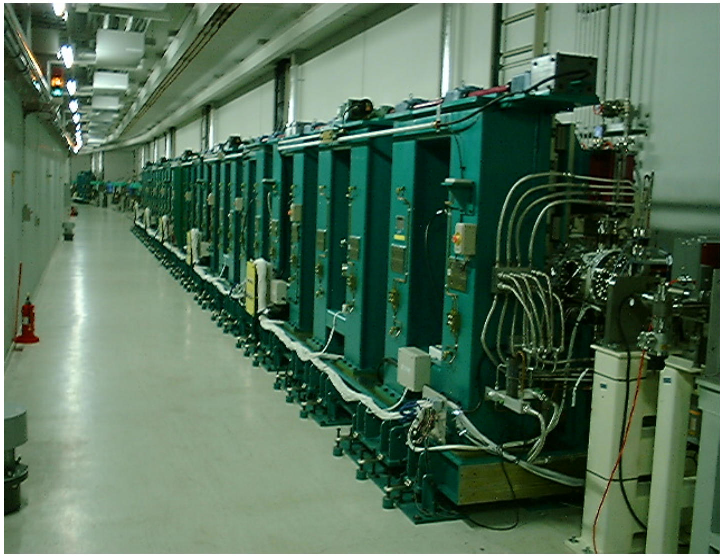
to keep the present experimental condition,
to improve the condition as much as possible, and
to challenge new experiments by using the advanced SR.

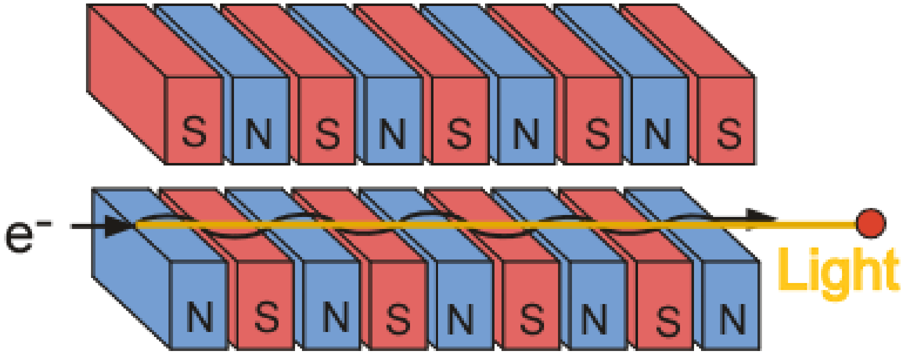
Outline

1. **Light source properties vs electron beam performance**
2. Stochasticity of photoemission
3. Distribution of circulating electron beam
4. Extra: Approach to coherent X-rays

2. Light source properties vs electron beam performance(1)

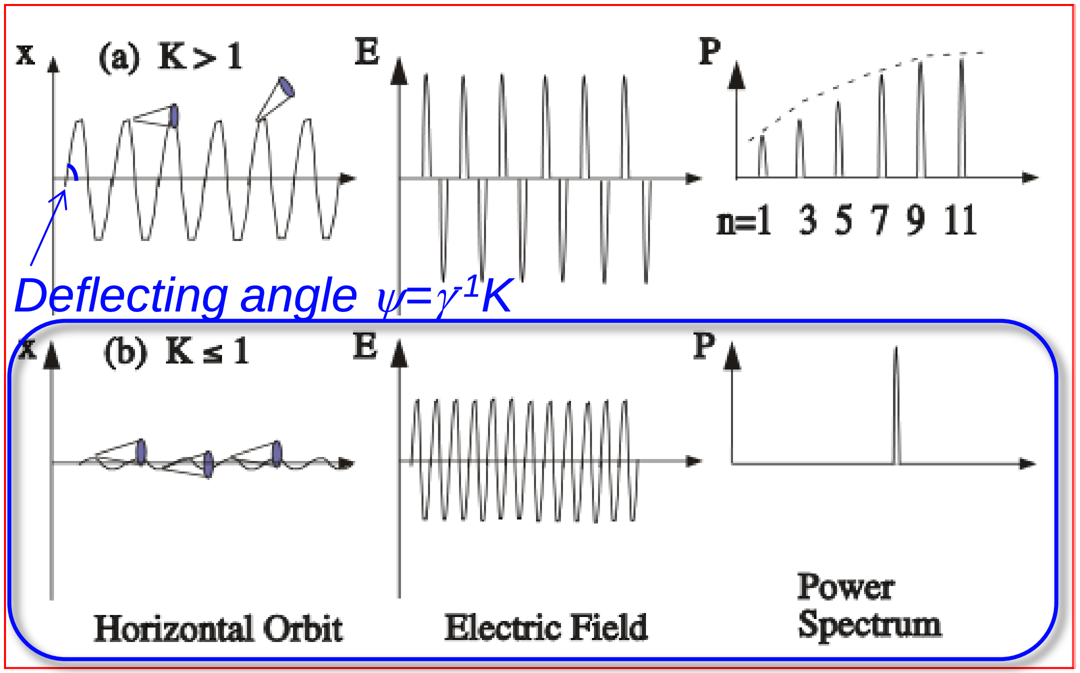
Main devices to supply SR to users are "undulators", which are installed in magnet-free straight sections in 3rd-generation SR sources.





2. Light source properties vs electron beam performance(2)

In an undulator, radiation from a single electron at each undulation interferes with each other.



Spectral narrowing by resonant condition for a single electron is

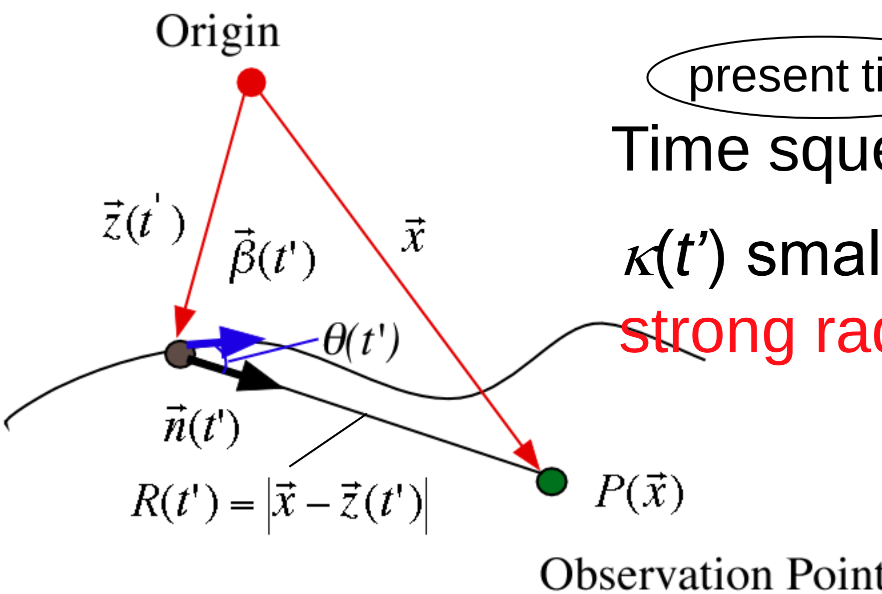
$$\lambda_1 = \frac{\lambda_u}{2\gamma^2}(1+K^2/2+\gamma^2\theta^2)$$

where K and γ are a deflecting parameter and relative energy.

2. Light source properties vs electron beam performance(3)

Q: Why does the radiation concentrate within the γ cone ?

A: The electron can run after the emitted light with the almost light speed only in this limited angle.



retarded time

present time

$$t = t' + \frac{R(t')}{c}$$

Time squeezing factor $\kappa(t') = \dfrac{\Delta t}{\Delta t'}$

$\kappa(t')$ small → large acceleration → strong radiation

$$\kappa(t') = \frac{\Delta t}{\Delta t'} = 1 - \vec{n}(t') \cdot \vec{\beta}(t')$$

$$\kappa(t') \approx \frac{1}{2}\left(\frac{1}{\gamma^2} + \theta^2\right)$$

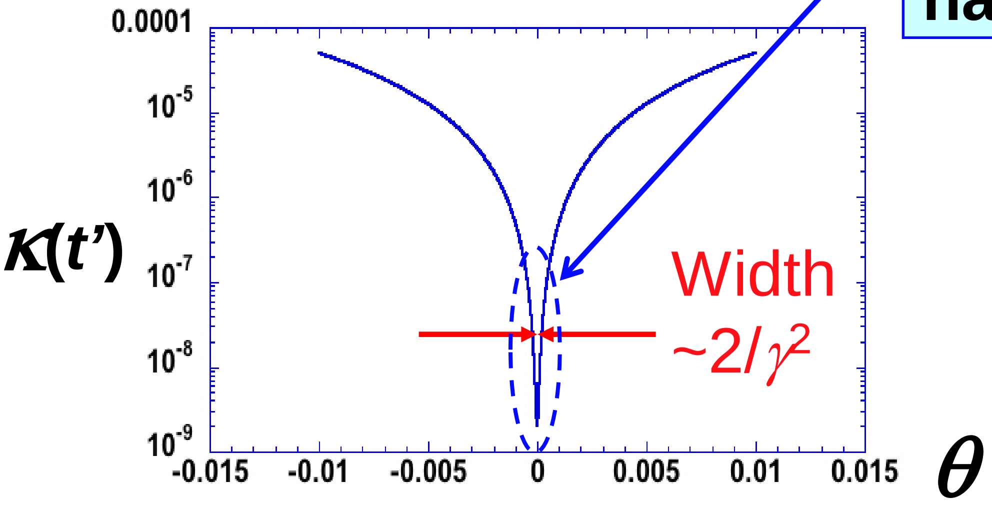
2. Light source properties vs electron beam performance(4)

Q: Why does the radiation concentrate within the γ cone ?

A: The electron runs after the emitted light with the almost light speed only in this limited angle.

$$\kappa(t') \approx \frac{1}{2}\left(\frac{1}{\gamma^2} + \theta^2\right) \qquad \theta < 1/\gamma \rightarrow \kappa \sim 1/\gamma^2$$

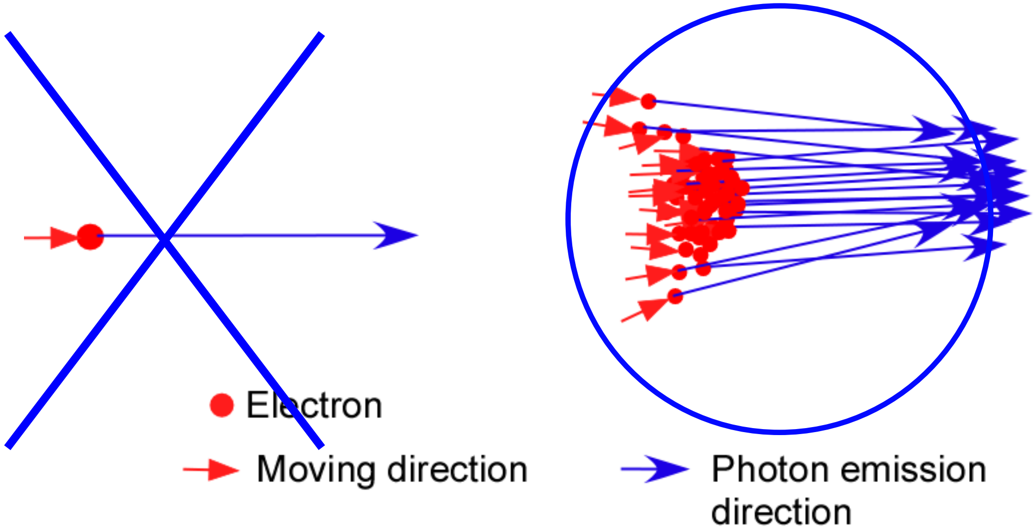
Intense radiation is emitted in this narrow angle



2. Light source properties vs electron beam performance(5)

If electron beam were point-like without spatial divergence and , all electrons could have no angular divergence, radiations were coherent !

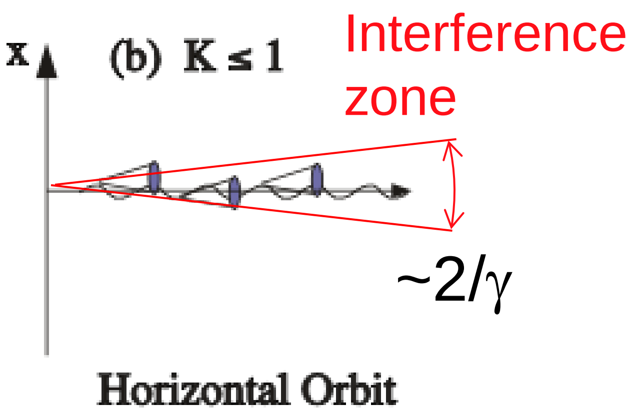
Real radiation properties are obtained by convoluting radiations from all N electrons

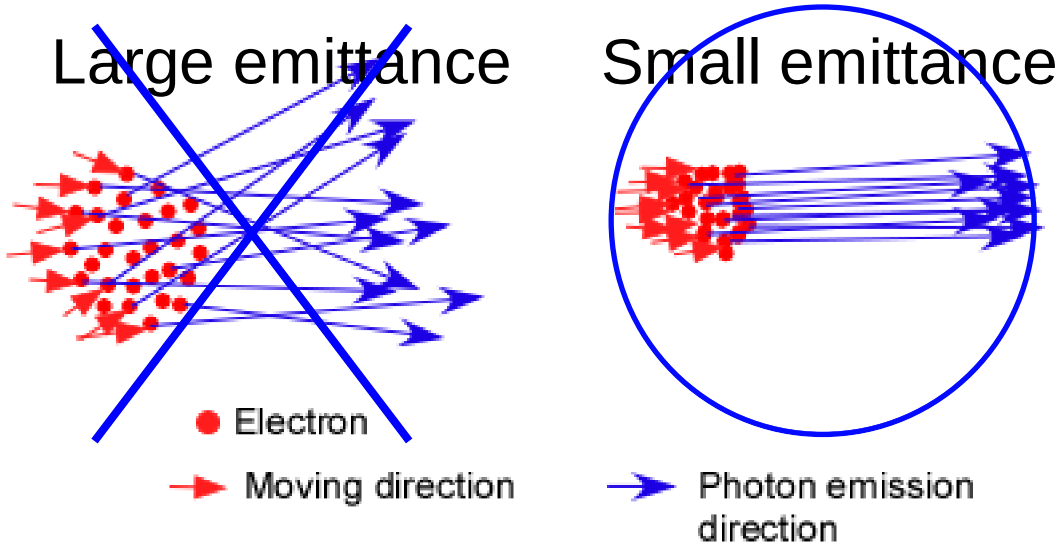


In the real world N electrons are distributed in a phase space, never degenerate on the same point.

2. Light source properties vs electron beam performance(6)

In order to obtain the same resonance condition over most of circulating electrons distributed, sufficiently small beam emittance is required.





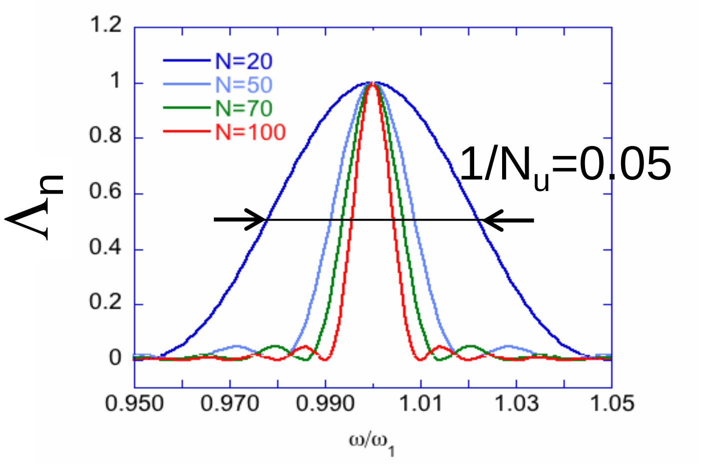
2. Light source properties vs electron beam performance(7)

In order to obtain the same resonance condition over most of circulating electrons distributed, sufficiently small beam energy spread is required.

$$\lambda_1 = \frac{\lambda_u}{2(\gamma+\Delta\gamma)^2}(1+K^2/2+\gamma^2\theta^2)$$

N_u:undulator period number

$$\Delta\lambda_1 \sim 2\lambda_1 <\Delta\gamma/\gamma>$$



Spectral broadening by the energy spread is much less than intrinsic broadening ~$1/N_u$.

2. Light source properties vs electron beam performance(8)

$$B \propto \frac{I_b \cdot N_u^{2} \cdot \gamma^2 \cdot F_n(K, \delta)}{\sigma_x \sigma_{x'} \sigma_y \sigma_{y'} \left(\sim \varepsilon_x \cdot \varepsilon_y \right)},$$

B: Brilliance (phs/sec/mm^2/mrad^2/100mA)

I_b: Beam current (mA)

N_u: Undulator period number

σ_x, σ_y: Horizontal and vertical beam sizes (m)

$\sigma_{x'}, \sigma_{y'}$: Horizontal and vertical angular divergence (rad)

δ: Beam energy spread

K :Deflection parameter

2. Light source properties vs electron beam performance(9)

$$\sigma_x = \sqrt{\sigma_p^2 + \beta_x \varepsilon_x + \eta_x^2 \delta^2}, \quad \sigma_{x'} = \sqrt{\sigma_{p'}^2 + \gamma_x \varepsilon_x + \eta'^2_x \delta^2},$$

$$\sigma_y = \sqrt{\sigma_p^2 + \beta_y \varepsilon_y + \eta_y^2 \delta^2}, \quad \sigma_{y'} = \sqrt{\sigma_{p'}^2 + \gamma_y \varepsilon_y + \eta'^2_y \delta^2},$$

$$\alpha_{x,y} = \frac{-d\beta_{x,y}}{ds}, \quad \gamma_{x,y} = \frac{1+\alpha_{x,y}^2}{\beta_{x,y}}, \quad \eta'_{x,y} = \frac{d\eta_{x,y}}{ds}.$$

Photon

Electron emittance

Electron energy spread

$\varepsilon_x, \varepsilon_y$: Horizontal and vertical emittance (m•rad)

$\beta_{x,y}$: Horizontal and vertical betatoron functions at ID

$\eta_{x,y}$: Horizontal and vertical dispersion functions at ID

$\sigma_p, \sigma_{p'}$: Spatial and angular divergence of photon beam

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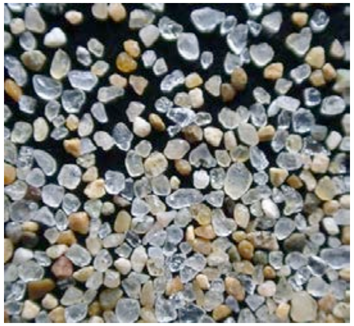
3. **Stochasticity of photoemission**
4. Distribution of circulating electron beam
5. Extra: Approach to coherent X-rays

3. Stochasticity of photoemission(1)

The photo-emission process is **not continuous**, in other words, **quantized process**. So, the emission position, the number of the emission photons, and the emission photon energy have **fluctuations**.

This stochasticity (random fluctuation) causes finite spread of the circulating electron beam in the 6D phase space. The density distribution is generally Gaussian due to the central limit theorem.



3. Stochasticity of photoemission(2)

A relativistic electron accelerated in a magnetic field will radiate electromagnetic energy at a rate which is proportional to the square of the accelerating force.

<Averaged radiation power>

classical electron radius (r_e)

$$P = \frac{2}{3}\frac{r_e c}{(m_0 c^2)^3} E^2 F_\perp^2 = \frac{2}{3}\frac{r_e c}{(m_0 c^2)^3}\frac{E^4}{\rho^2}\ .$$

<Averaged radiation energy per turn>

$$U = \int^T P\,dt = \int^L P \frac{d\ell}{c},$$

$$U(\mathrm{keV}) = \frac{88.5 E^4(\mathrm{GeV})}{\rho(\mathrm{m})}$$ for the case with the constant ρ.

Averaged properties are smooth !

3. Stochasticity of photoemission(3)

In the quantized radiation process, the integrated parameter, energy loss per unit time also fluctuates. Magnitude of the fluctuation can be defined by a mean square.

<Mean square of energy loss fluctuation per unit time>

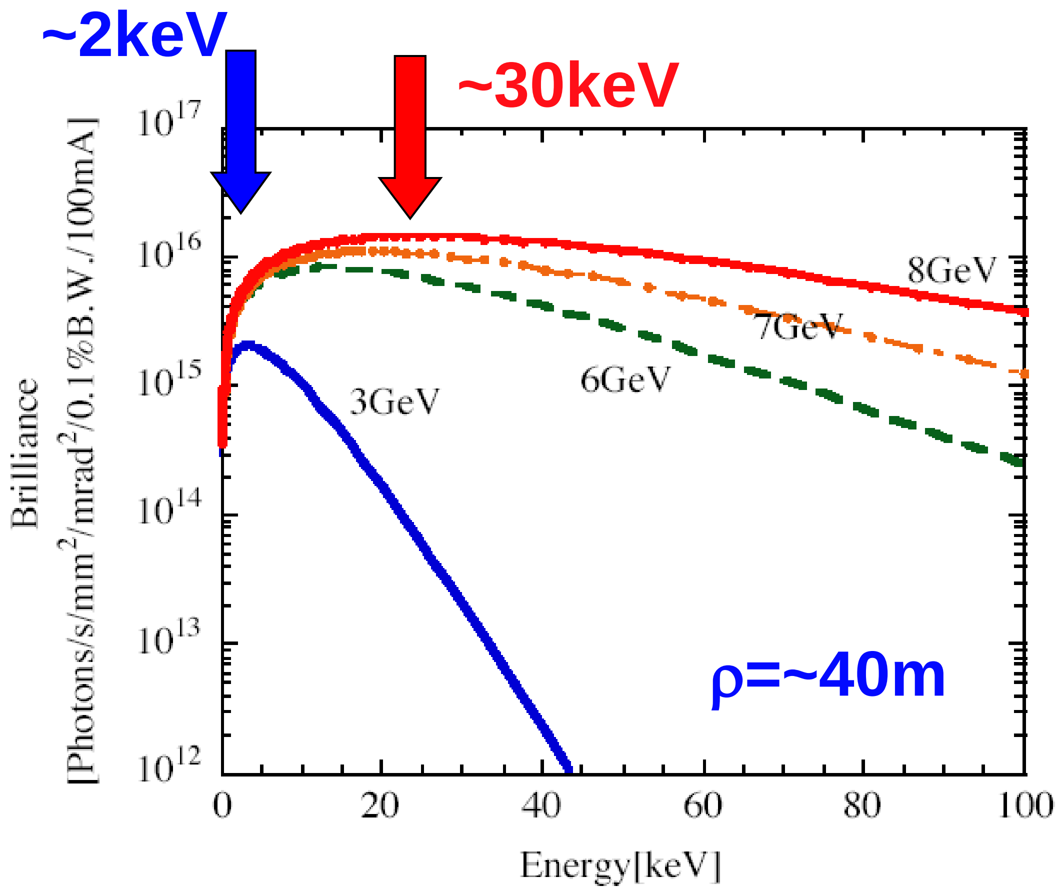
$$\langle Nu^2 \rangle = \frac{55}{24\sqrt{3}} \frac{U}{T} u_c \, , \qquad u_c = \frac{3}{2} \frac{\hbar c}{\rho} \gamma^3 \, .$$

u_c: critical photon energy ~ $3.2\langle u \rangle$

<Averaged photo-emission rate>

$\langle N \rangle$(phs/s) ~ $3.2P/\, u_c$

3. Stochasticity of photoemission(4)



u_c represents the photon energy at the peak brilliance of the BM radiation.

3. Stochasticity of photoemission(5)

Let's estimate the parameters!

<Case-1 E=1 GeV, ρ=5m>

$$U(\text{keV}) = \frac{88.5\times 1^4}{5} = 17.7, \quad P(\text{keV/s}) = \frac{U}{T=2\pi\times 5/c} = 1.7\times 10^8$$

$$u_c\,(\text{keV}) = \frac{3}{2}\,\frac{6.85\times 10^{-19}\times 2.998\times 10^8\times 1957^3}{5} = \mathbf{0.44}$$

$$\langle Nu^2\rangle\,(\text{keV}^2/\text{s}) = \frac{55}{24\sqrt{3}}\,1.7\times 10^8 \times 0.44 = 0.99\times 10^8$$

$$\langle N\rangle(\text{photons/s}) \sim 3.2\times 1.7\times 10^8 \,/\, 0.44 = \mathbf{1.2\times 10^9}$$

3. Stochasticity of photoemission(6)

SPring-8 parameter is the next example.

<Case-2 E=8 GeV, ρ=40m>

$$U(\mathrm{keV}) = \frac{88.5\times 8^4}{40} = 9.1\times 10^3,\quad P(\mathrm{keV/s}) = \frac{U}{T=2\pi\times 40/c} = 1.1\times 10^{10}$$

$$u_c\ (\mathrm{keV}) = \frac{3}{2}\ \frac{6.85\times 10^{-19}\times 2.998\times 10^8\times 15656^3}{40} = \mathbf{29.6}$$

$$\langle Nu^2\rangle\ (\mathrm{keV}^2/\mathrm{s}) = \frac{55}{24\sqrt{3}} 1.1\times 10^{10}\times 29.6 = 4.31\times 10^{11}$$

$$\langle N\rangle(\mathrm{photons/s}) \sim 3.2\times 1.1\times 10^{10} / 29.6 = \mathbf{1.2\times 10^9}$$

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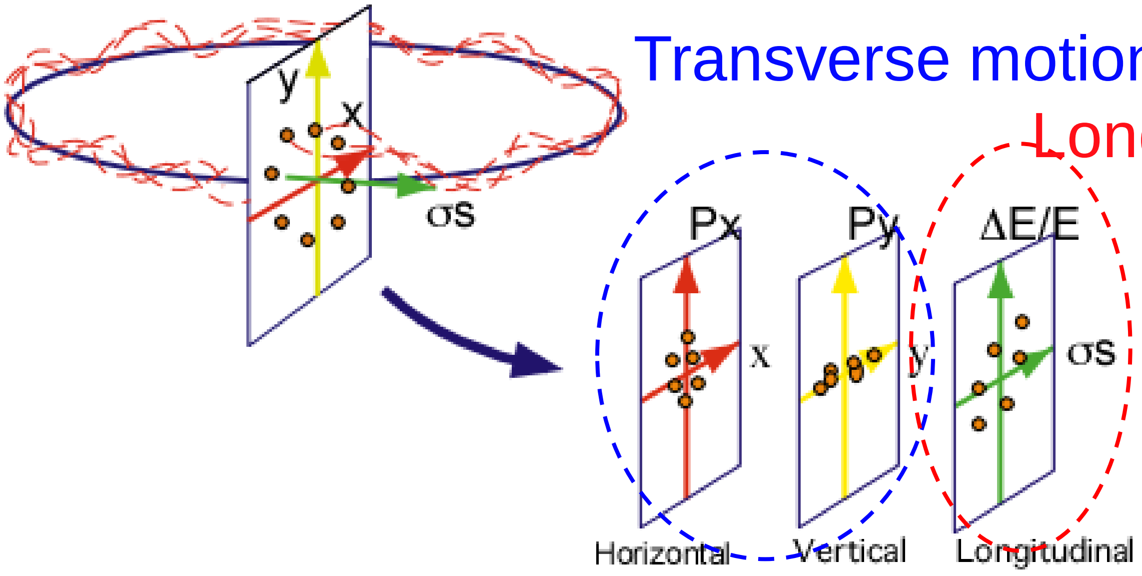
4. Distribution of circulating electron beam(1)

6D-phase space volume of a single electron, ε_{s6} comprises of canonical variables (x, px(x'), y, py(y'), t, ps(ΔE/E)).
In an ideal case, 6D-phase space volume can be written by the product of areas of the three orthogonal 2D spaces ε_{sz}.

z=x,y,s



$$\varepsilon_{s6} = \varepsilon_{sx}\varepsilon_{sy}\varepsilon_{ss}$$

These are invariants of the motion for a conservative system.

4. Distribution of circulating electron beam(2)

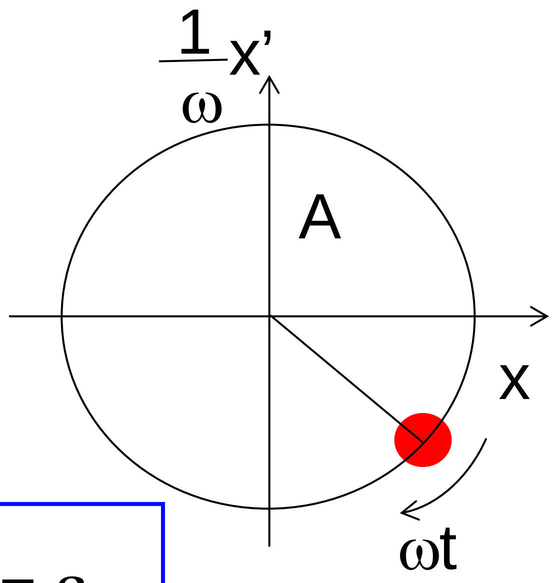
You can understand the relation between the 2D phase space and the emittance by using a simple harmonic oscillator.

$$x = A \cdot \cos(\omega t + \phi_0)$$

$$\frac{dx}{dt} = x' = -A \cdot \omega \sin(\omega t + \phi_0)$$

$$\longrightarrow \quad \frac{1}{\omega}\frac{dx}{dt} = -A \cdot \sin(\omega t + \phi_0)$$

Action=A, Angle= $\phi = \omega t + \phi_0$,



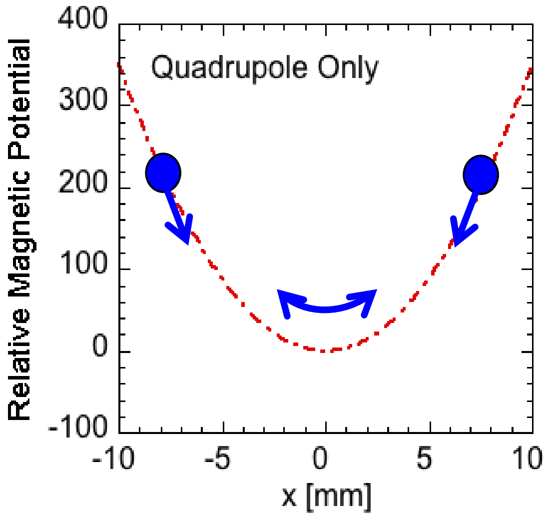
$$\text{Invariant} = x^2 + (x'/\omega)^2 = A^2 = \frac{\text{circle area}}{\pi} = \varepsilon_{sx}$$

4. Distribution of circulating electron beam(3)

We need potential wells to stabilize three orthogonal oscillation modes to keep electron beam in a storage ring.

Quadrupole magnets generate the adequate potential wells for two transversal oscillation modes, which are called betatron oscillations in the horizontal and vertical planes.

RF acceleration electric field generates the adequate potential well for longitudinal oscillation mode, which is called a synchrotron oscillation.



4. Distribution of circulating electron beam(4)

SR light properties reflects the 3×2D phase space distribution of circulating electrons.

We use the following **three** **ensemble-averaged emittances** to express beam distribution in three orthogonal phase spaces.

$$<\varepsilon_{s6}> = <\varepsilon_{sx}><\varepsilon_{sy}><\varepsilon_{ss}>$$
$$=\varepsilon_x \varepsilon_y \varepsilon_s,$$

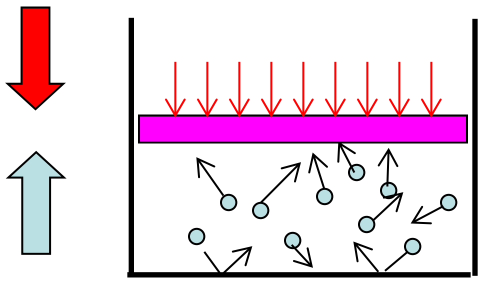
ε_x : horizontal emittance (m rad)

ε_y : vertical emittance (m rad)

ε_s : longitudinal emittance (m rad)

4. Distribution of circulating electron beam(5)

Width of the Gaussian distribution of N circulating electrons is determined by the **dynamical equilibrium between the radiation excitation and damping**.



4. Distribution of circulating electron beam(6)

Remember the invariant of a harmonic oscillator.

$$\frac{d\langle A^2\rangle}{dt} = \frac{d\varepsilon_z}{dt} \qquad z = x,y,s.$$

Equilibrium condition:

$$\frac{d\langle A^2\rangle}{dt} = \lim_{\Delta t \to 0} \frac{\langle (A+\Delta A)^2 - A^2\rangle}{\Delta t} = \lim_{\Delta t \to 0} 2\frac{\langle A\Delta A\rangle}{\Delta t} + \lim_{\Delta t \to 0} \frac{\langle \Delta A^2\rangle}{\Delta t} = 0$$

Damping term
Averaged energy dissipation

Excitation term
Quantum effect

4. Distribution of circulating electron beam(7)

Energy spread $\sigma_{\Delta E}$

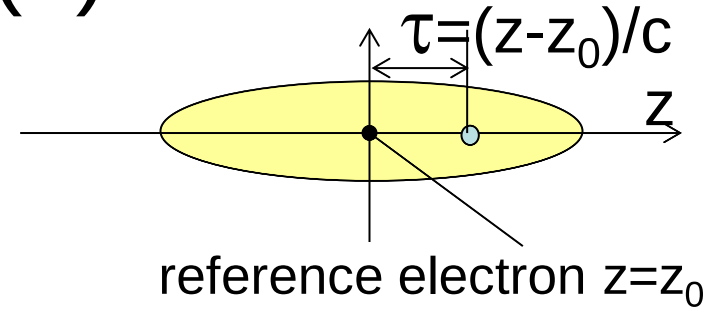
$$\lim_{\Delta t \to 0} 2\frac{<A\Delta A>}{\Delta t} = -2\frac{<A^2>}{\tau_\varepsilon}\ , \qquad \tau_\varepsilon = \frac{E \times T}{U}\ ,$$

$$\lim_{\Delta t \to 0} \frac{<\Delta A^2>}{\Delta t} = <Nu^2> = \frac{55}{24\sqrt{3}}\frac{U}{T}u_c\ .$$

Since $\sigma_{\Delta E}$ is not the emittance, phase average factor should be considered, $\sigma_{\Delta E}{}^2 = \frac{<A^2>}{2}$,

$$\sigma_{\Delta E} = \sqrt{\frac{55}{96\sqrt{3}}E \times u_c}\ , \quad \sigma_{\Delta E/E} = \sqrt{\frac{55}{96\sqrt{3}}\frac{u_c}{E}}\ .$$

4. Distribution of circulating electron beam(8)



Bunch length σ_τ

σ_τ has two components;

$\sigma_{\tau 1}$= time spread by energy spread

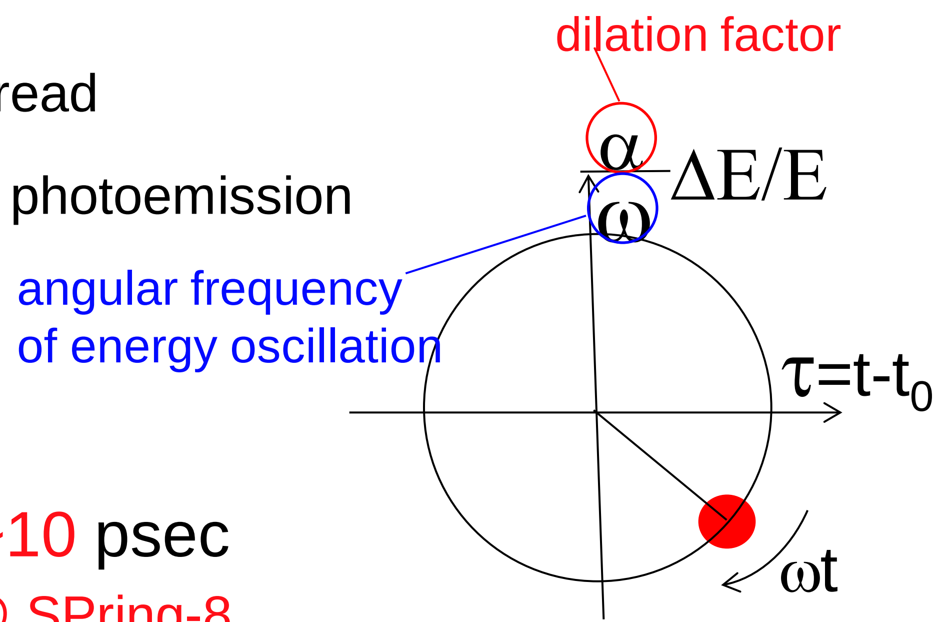
$\sigma_{\tau 2}$= time spread by stochastic photoemission

$$\sigma_{\tau 1} >> \sigma_{\tau 2}$$

$$\sigma_\tau = \frac{\alpha}{\omega}\sigma_{\Delta E/E} = \sim 10 \text{ psec}$$

(α: 10^{-4}; $\sigma_{\Delta E/E}$: 10^{-3}; ω: 10^{4})

@ SPring-8



Phase space of synchrotron (energy) oscillation

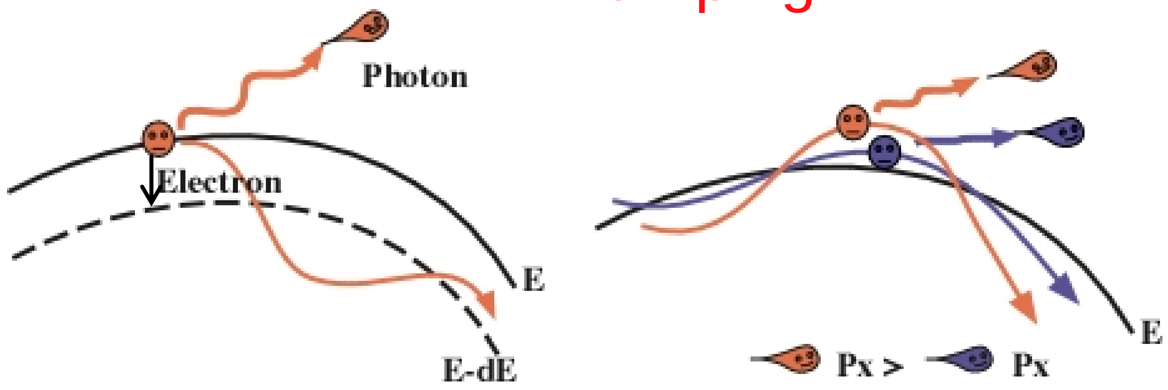
4. Distribution of circulating electron beam(9)

Horizontal emittance ε_x

Excitation: due to the discrete energy jump + energy dispersion

Damping: due to the decrease of transverse momentum by the photoemission + acceleration along the running direction

Excitation Damping



4. Distribution of circulating electron beam(10)

For the typical magnetic lattice structure (Chasman Green: CG) based storage ring, the horizontal minimum emittance is written by

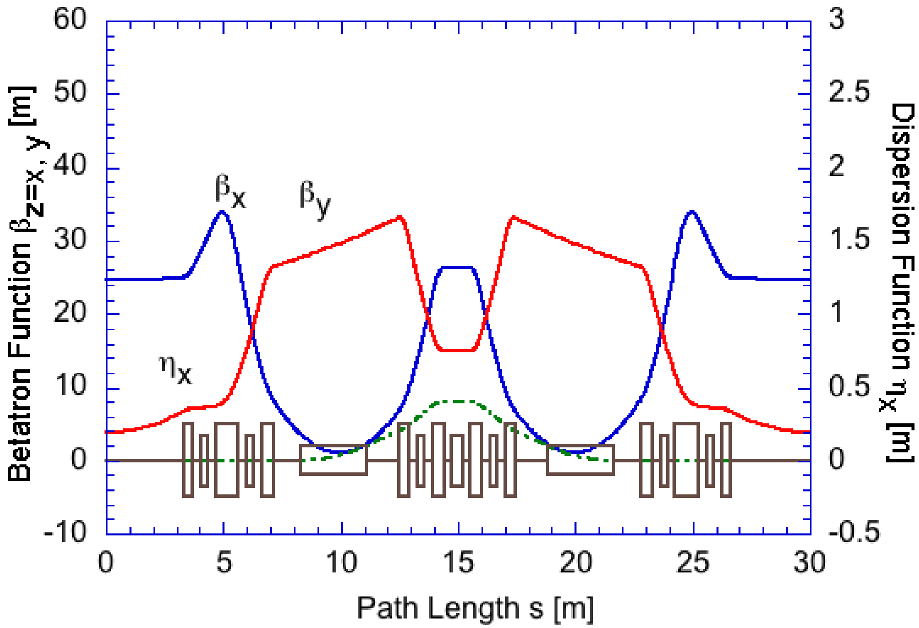
$$\varepsilon_{x\,\min}\ \text{@general} \sim \frac{1}{2}\varepsilon_{x\,\min}\ \text{@achromat}$$

$$= \frac{C_q \gamma^2}{8\sqrt{15}\ J_x}\theta_b^{\,3}$$

C_q: Quantumn constant 3.832×10^{-13} (m)
θ_b: Deflection angle of a single bending magnet (rad)
Jx: Horizontal damping partition number ~ 1

4. Distribution of circulating electron beam(11)

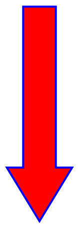
Chasman-Green (CG) lattice is the most popular magnet cell structure for a low emittance SR souce, where a achromatic arc is composed of a pair of bending magnets.



2 bends: CG or CBA

3 bends: TBA

4 bends: QBA



$\varepsilon_{x\ min} \propto \theta_b^3$

θ_b smaller with same cell No.

4. Distribution of circulating electron beam(12)

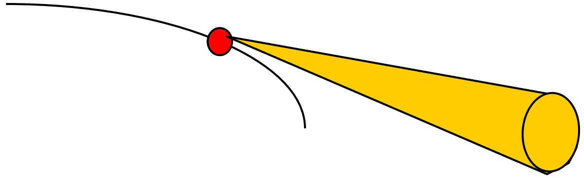
Vertical emittance ε_y

ε_y has two components;

ε_{y1}= vertical emittance by stochastic photoemission

ε_{y2}= vertical emittance by HV coupling

Usually, $\varepsilon_{y1} << \varepsilon_{y2}$



The Angular divergence
In the vertical plane is ~ $1/\gamma$

1-GeV storage ring

$\gamma=1000/0.511\sim2000$

$1/\gamma\sim5\times10^{-4}$

Principally, $\varepsilon_y = 1/1000\ \varepsilon_x$ is possible

4. Distribution of circulating electron beam(13)

Vertical emittance is determined by magnetic error components mixing the horizontal and vertical betatron oscillation. The main sources are vertical misalignments of sextupole magnets and rotational errors of quadrupole magnets.

The effect of these error fields can be corrected to **0.1 % level** by the combination of beam response analysis and skew quadrupole corrector magnets.

One-dimensional diffraction limited X-ray beam is now available

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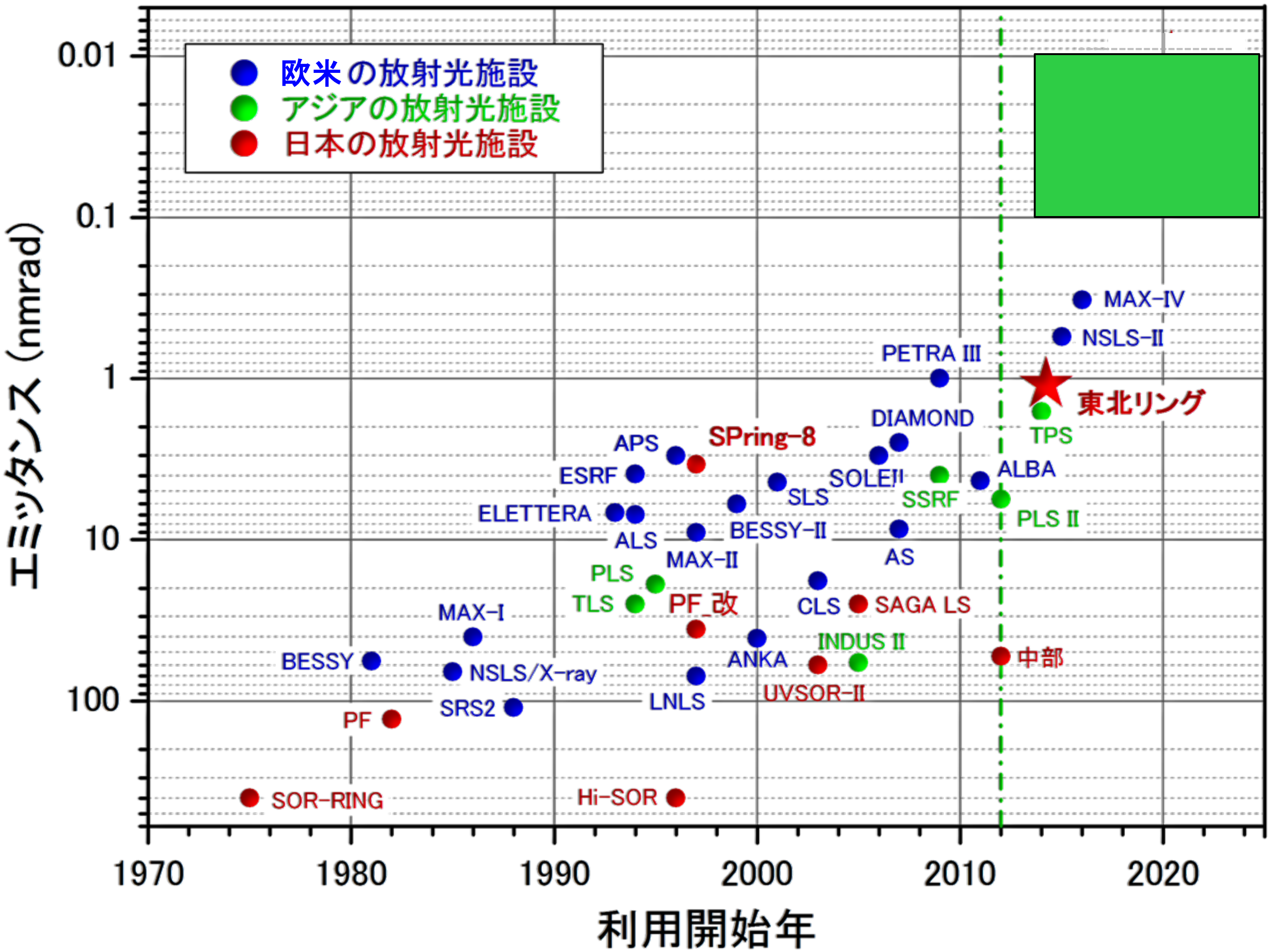
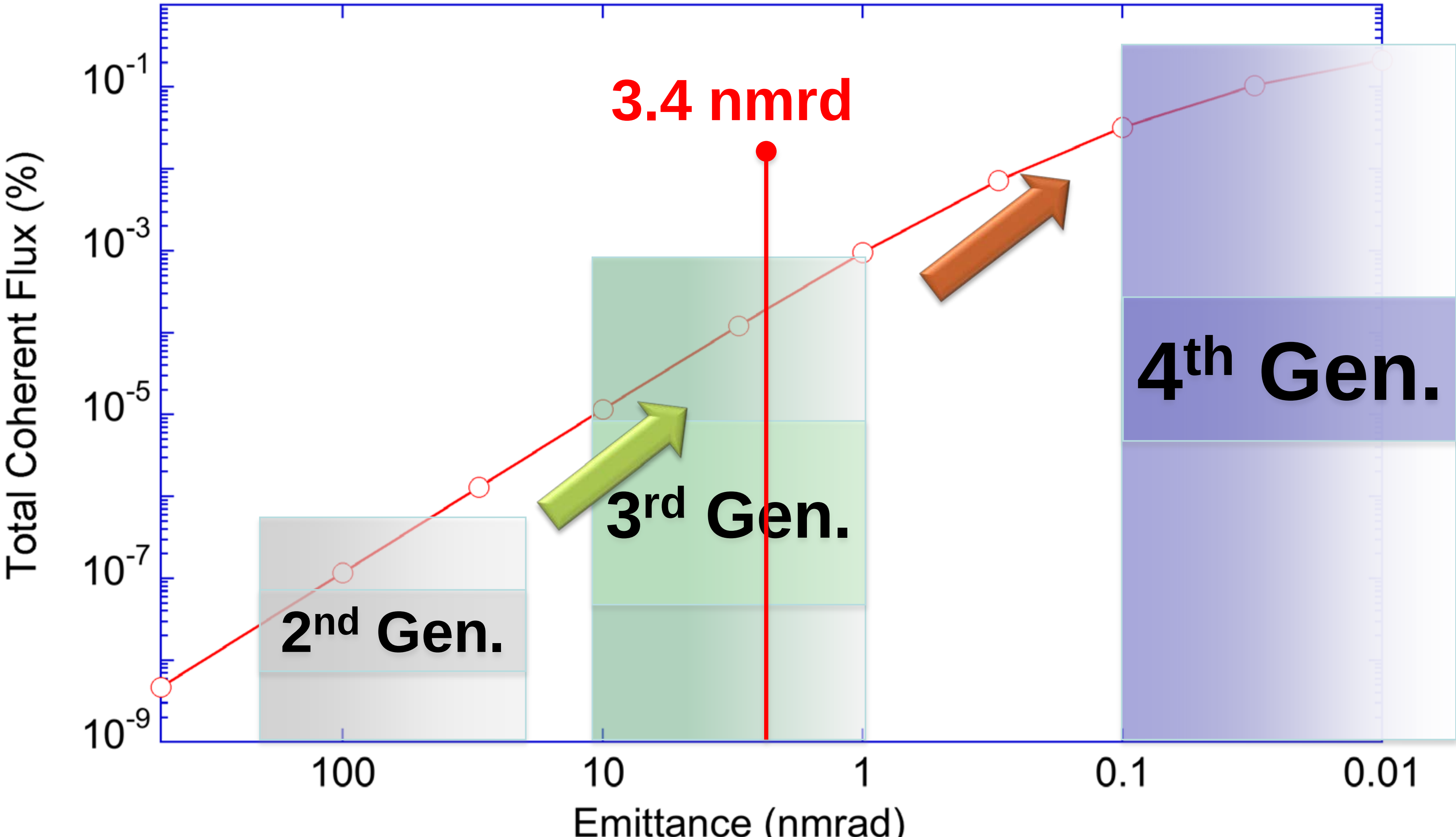
5. Approach to coherent X-rays (1)



5. Approach to coherent X-rays (2)



5. Approach to coherent X-rays (3)

Equation of natural emittance:

$$\varepsilon_{nat} = C_q \frac{\gamma^2 \langle H/\rho^3 \rangle}{J_x \langle 1/\rho^2 \rangle} \propto \frac{\gamma^2 \theta^3}{J_x}$$

γ : **Lorentz factor**
θ : **Bending angle**
ρ : **Bending radius**
H : **H-function**
J_x : **Damping partition number**

Conventional reduction scheme:

1. **Reduction of bending angle (θ) by increasing the number of bending magnets**

Additional reduction schemes:

2. **Reduction of stored energy (γ) with the help of advanced undulator design**
3. **Optimization of dipole field (ρ) in a dipole and / or inside unit cell)**
4. **Damping enhancement ($<H/\rho^3>/<1/\rho^2>$) by additional radiation**
5. **Damping partition number (J_x) control**