Light Sources I

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Outline

- Introduction
- Fundamentals of Light and SR
- Overview of SR Light Source
- Characteristics of SR (1)
- Characteristics of SR (2)
- Practical Knowledge on SR

Lecture I? (Introduction; Fundamentals of Light and SR)

Lecture II? (Overview of SR Light Source; Characteristics of SR (1); Characteristics of SR (2); Practical Knowledge on SR)

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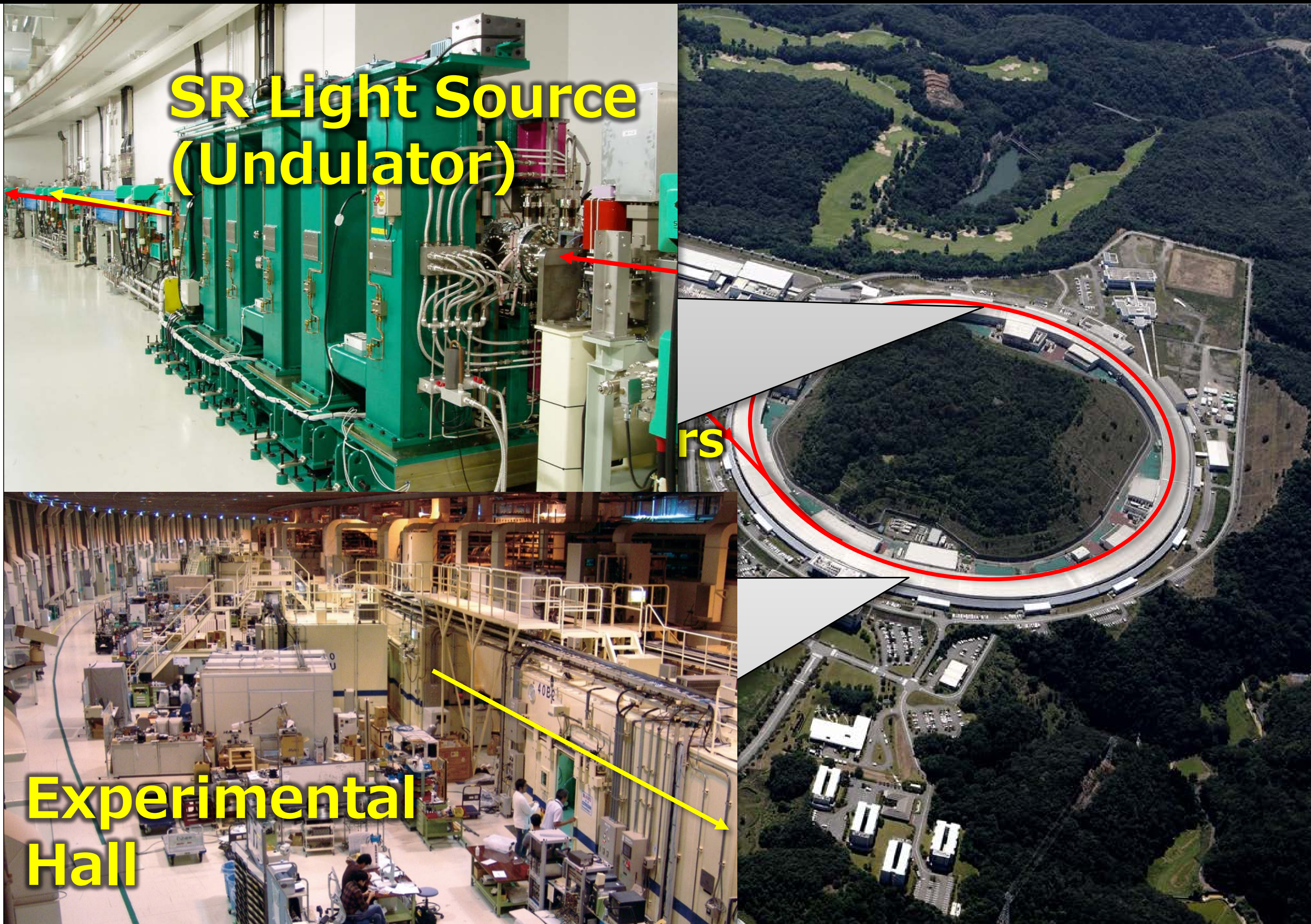
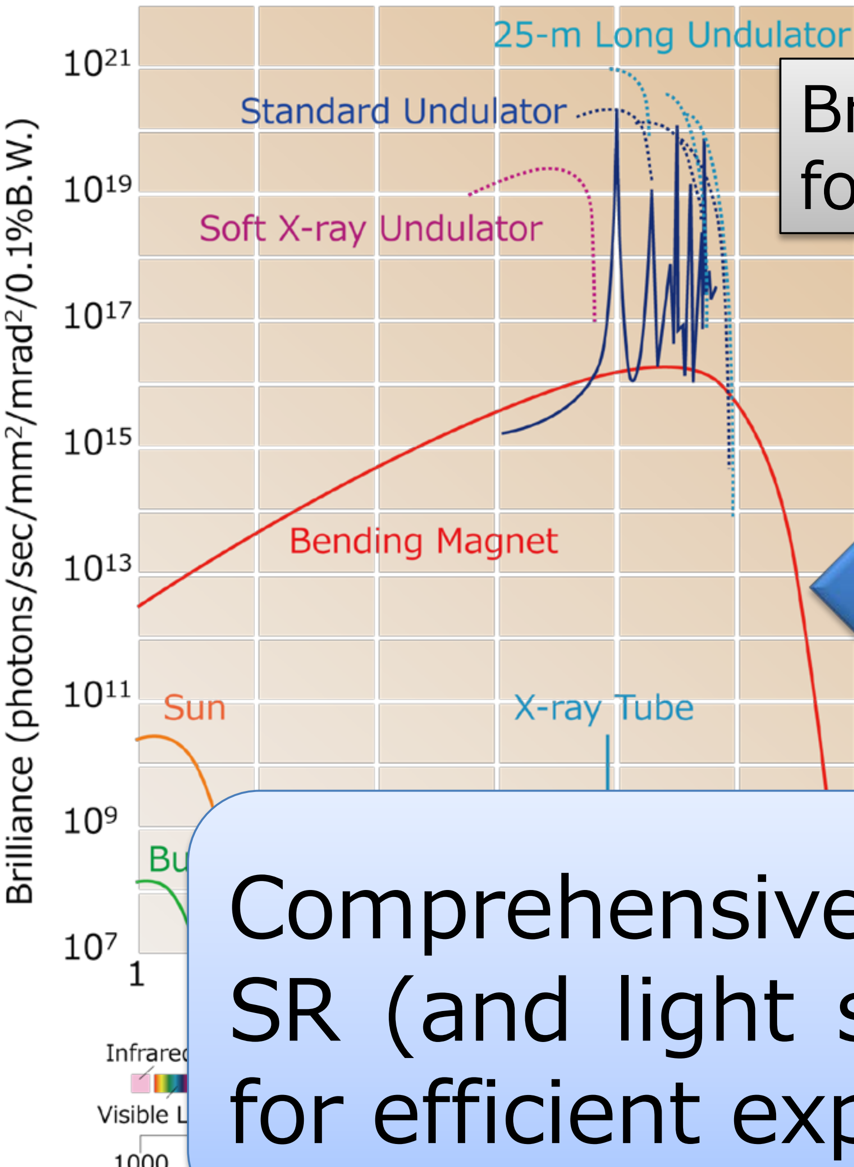
Overview of SR Facility



What's the Advantage of SR?



Brilliance vs. wavelength for various light sources

Light Source	**Total Power (W)**
Sun	10^{26}
X-ray Tube	10^{3}
Light Bulb	10^{2}
SR	**$10^{3}\sim10^{4}$**

Comprehensive understanding of SR (and light source) is required for efficient experiments.

Topics in This Lecture (1)

- Fundamentals of Light and SR
  - General description of light
  - Why we need SR?
  - Physical quantity of light
  - Uncertainty of light: Fourier and diffraction limits
  - SR: Light from a moving electron
- Overview of SR Light Source
  - Types of light sources
  - Magnet configuration

Topics in This Lecture (2)

- Characteristics of SR
  - Radiation from bending magnets
  - Electron Trajectory in insertion devices
  - Radiation from insertion devices
- Practical Knowledge on SR
  - Finite emittance and energy spread
  - Heat load and photon flux
  - Evaluation of optical properties of SR
  - Definition of undulators and wigglers
  - Numerical examples

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What is Light?

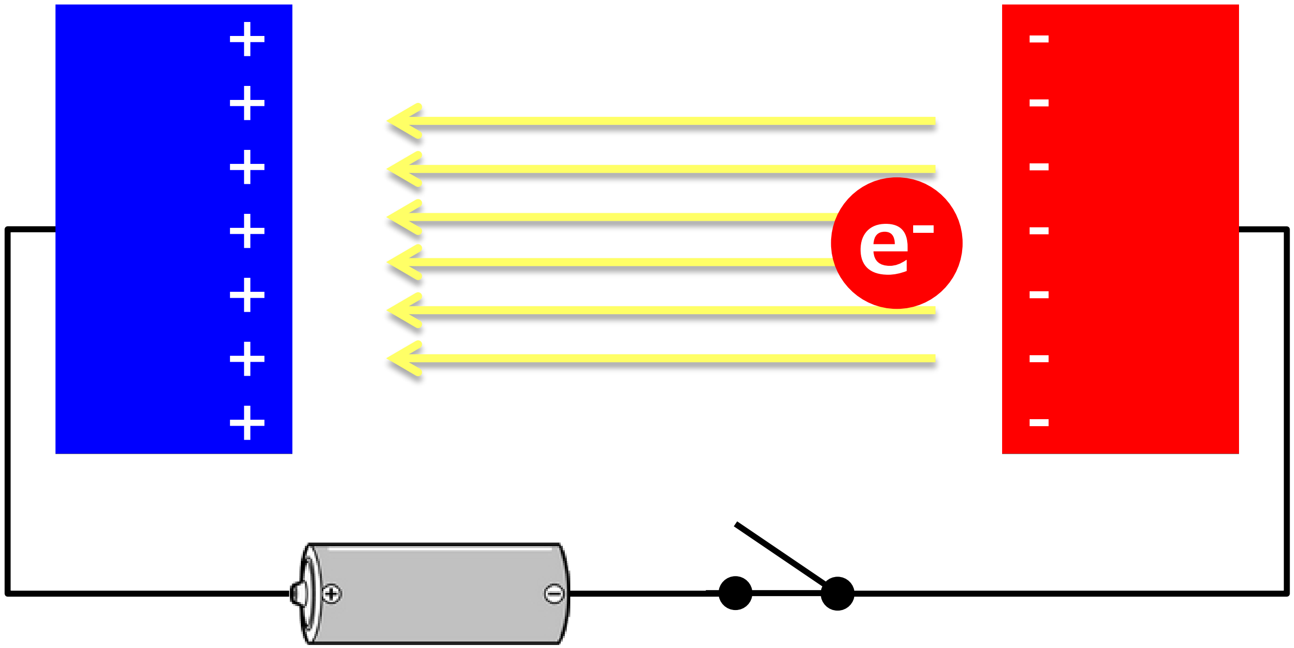
What is light? It is a kind of wave, but…

Various Waves

What propagates the light wave?

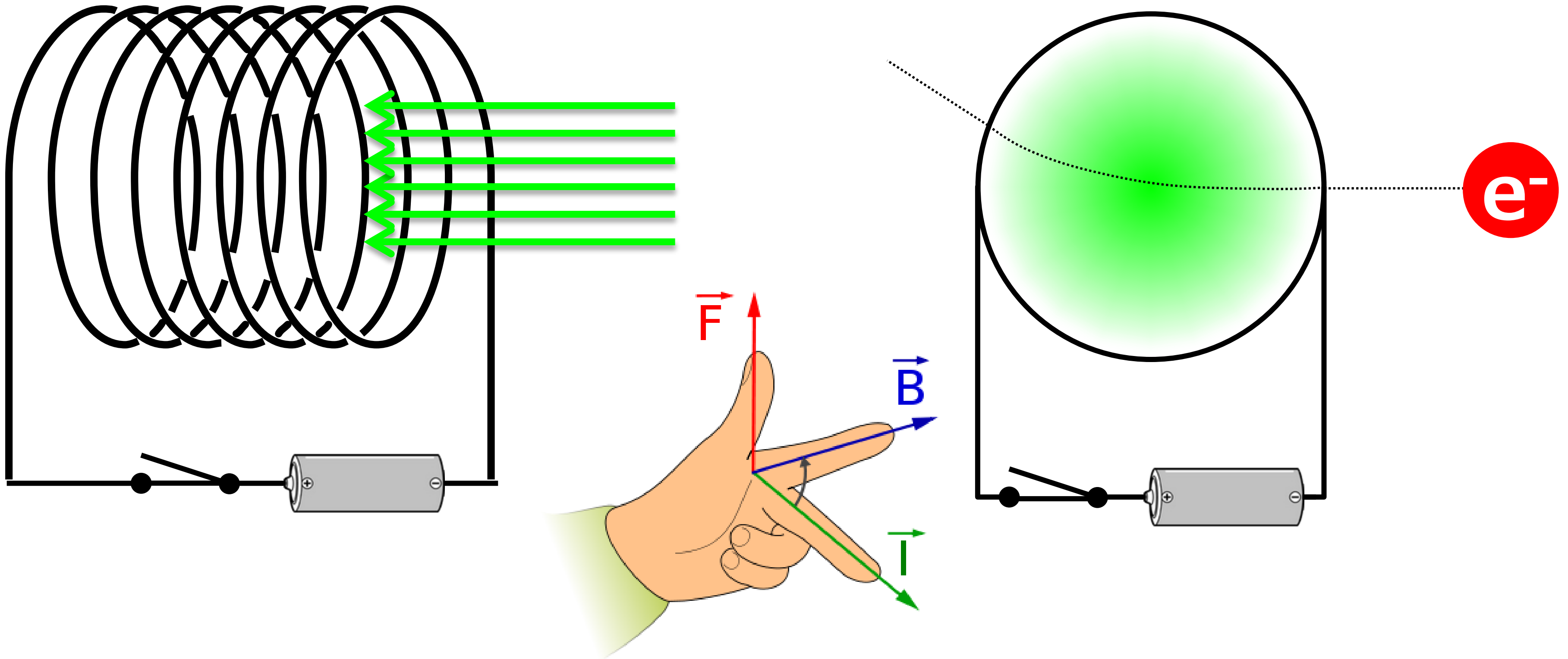
➡Electric and Magnetic Field

Electric Field



The E-field is generated by electric charges, and gives a force on a charged particle.

Magnetic Field



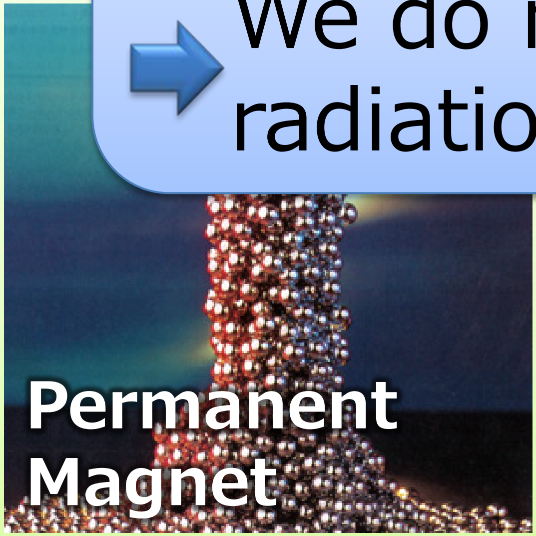
The M-field is generated by moving electric charges, and give a force on a moving charged particle.

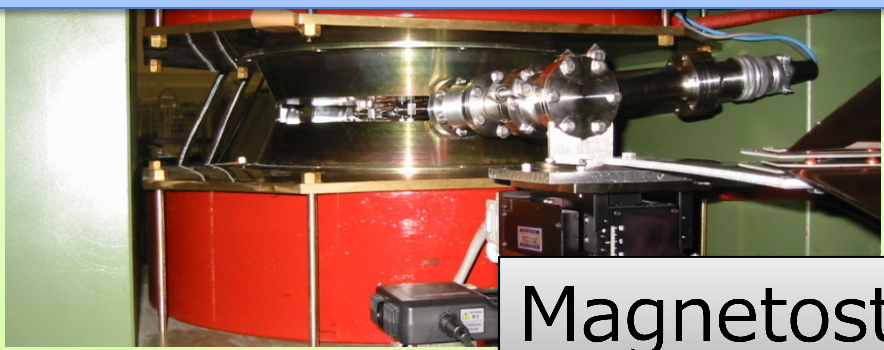
Electro- and Magnetostatic Phenomena

Elect

Motions of electrons (charges) are stationary or steady.

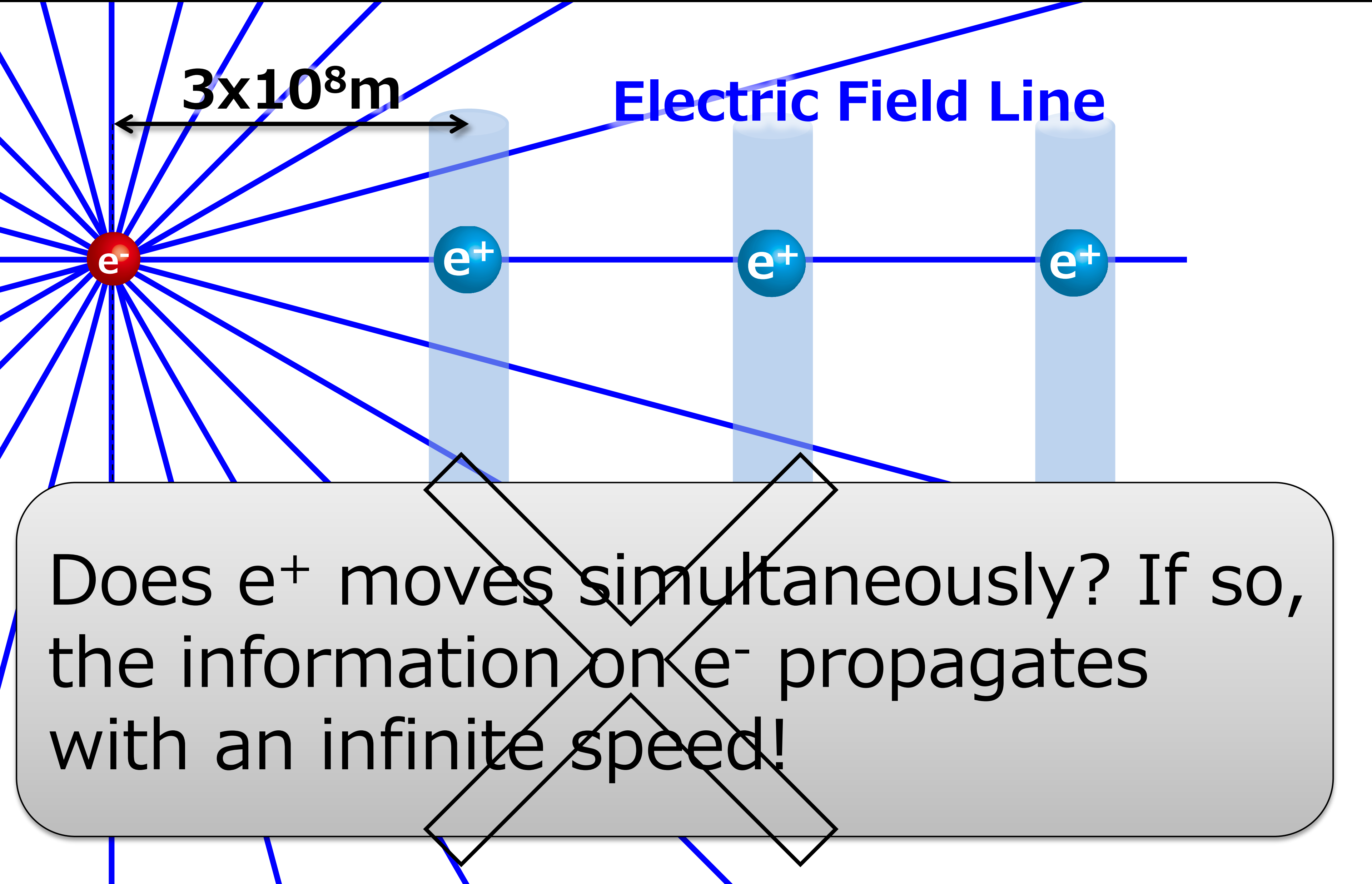
➡ We do not observe any radiation in such a steady state.





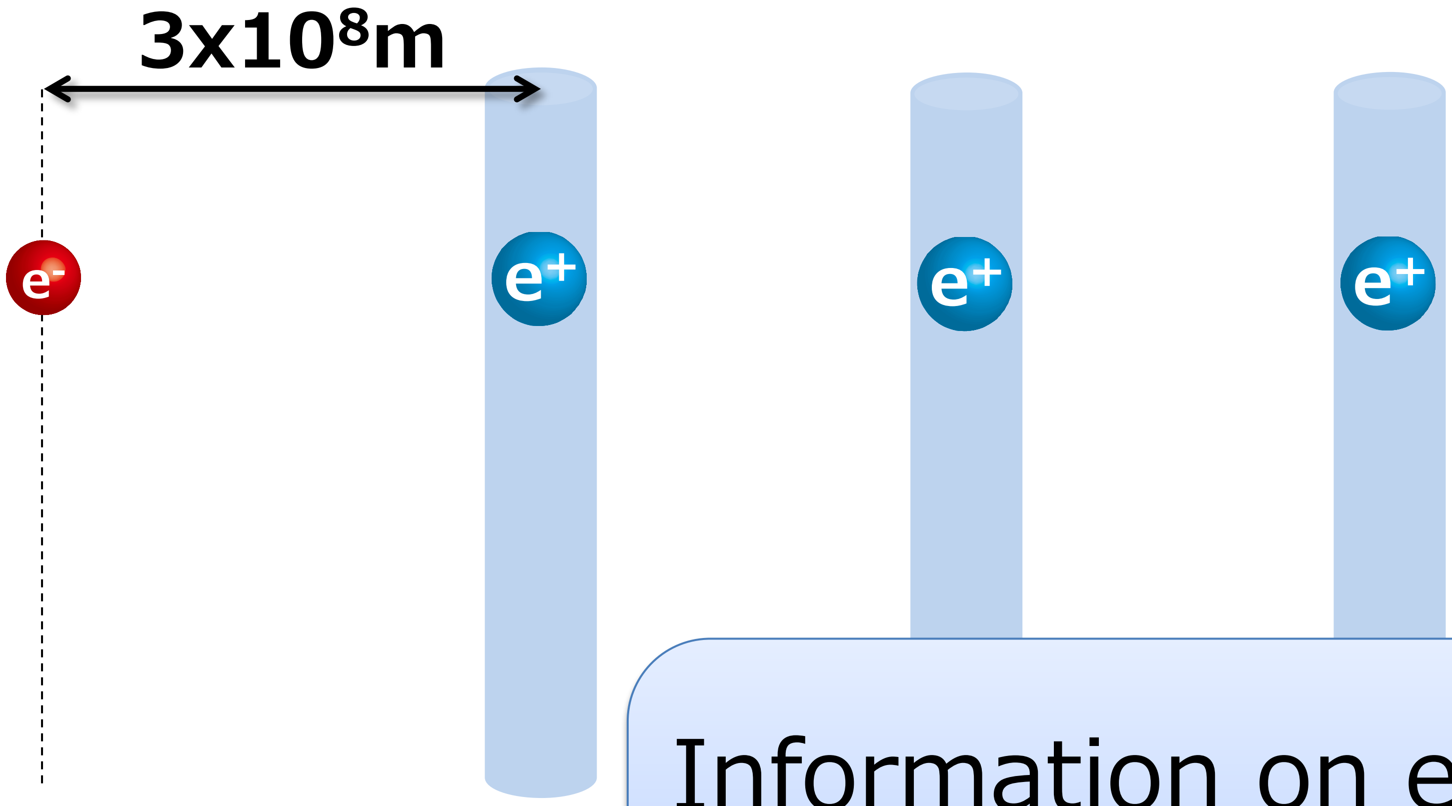
Magnetostatic

Thought Experiment



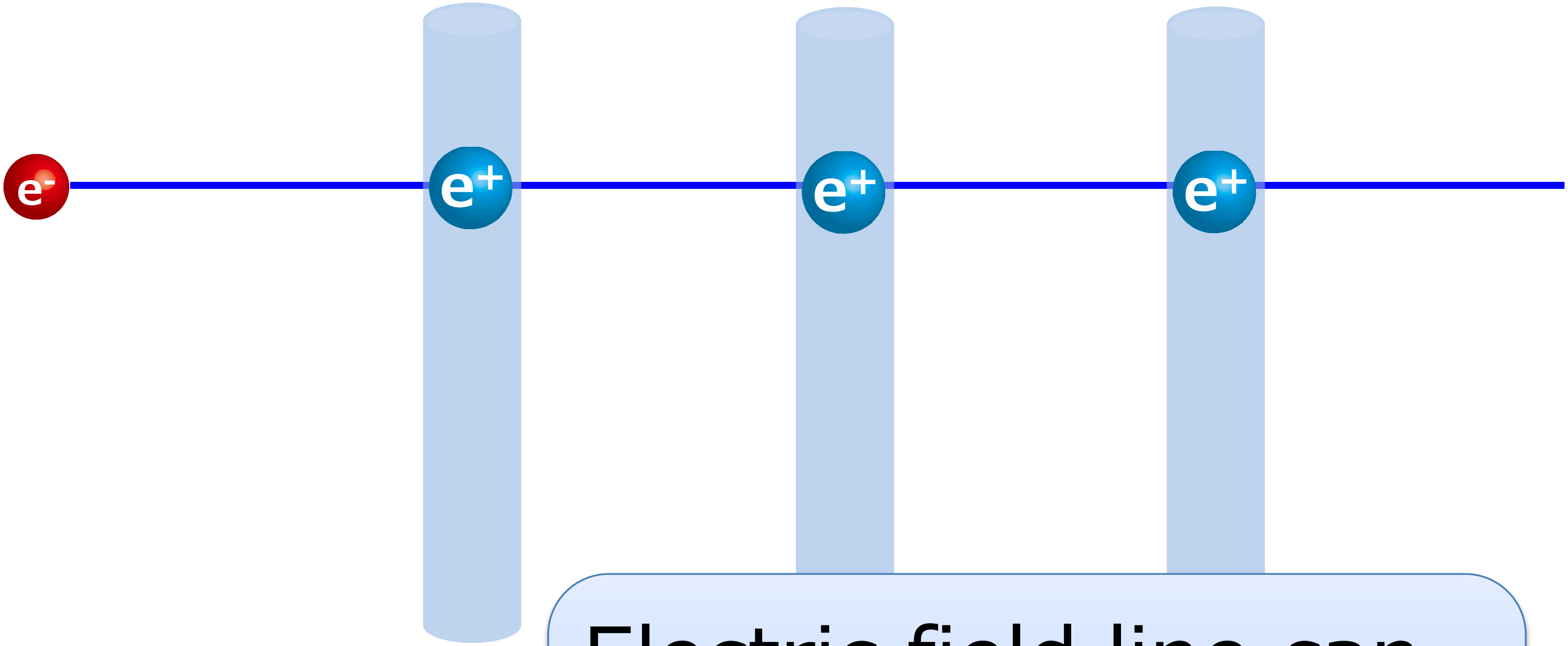
Does e^+ moves simultaneously? If so, the information on e^- propagates with an infinite speed!

Thought Experiment



Information on e^- (E-field) propagates with the speed of light!

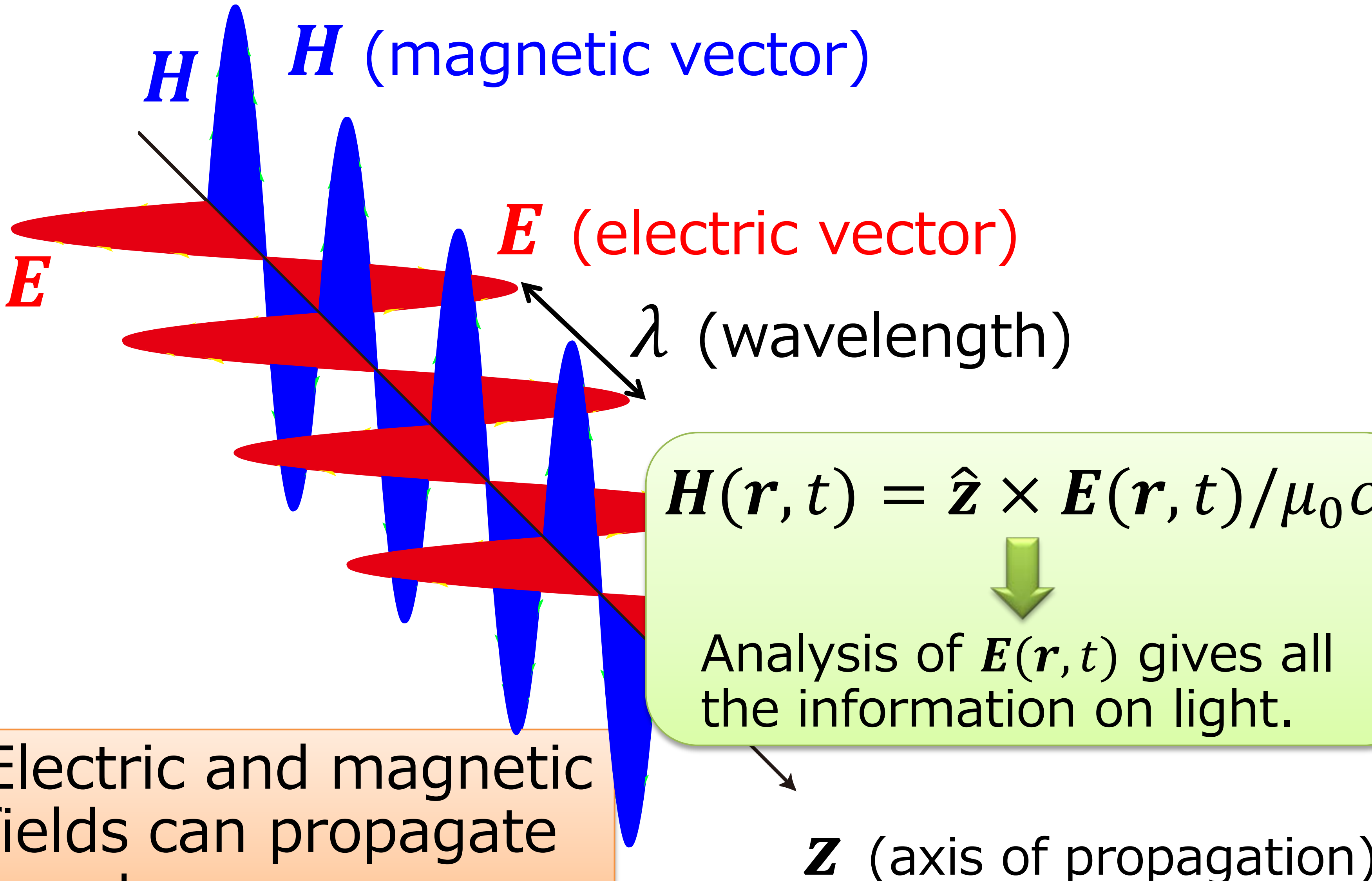
E-Field Line Is Not "Rigid"



Electric field line can propagate as a wave

➡ **Source of Light**

Light as an Electromagnetic Wave



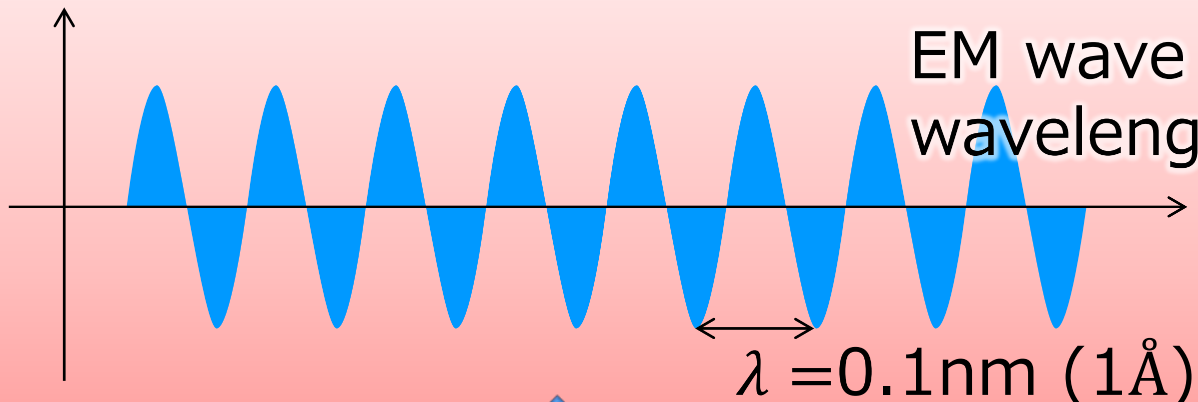
$$\boldsymbol{H}(\boldsymbol{r},t) = \hat{\mathbf{z}} \times \boldsymbol{E}(\boldsymbol{r},t)/\mu_0 c$$

Analysis of $\boldsymbol{E}(\boldsymbol{r},t)$ gives all the information on light.

Electric and magnetic fields can propagate as a transverse wave

Light as a Photon

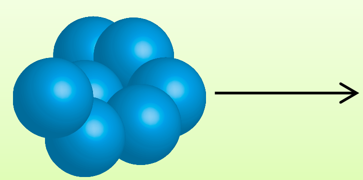
Light is not only an electromagnetic wave but also a particle, or a photon.



EM wave with a wavelength of 0.1nm

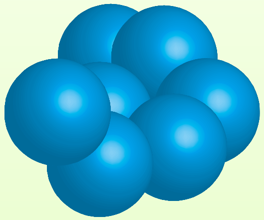
$$E_{photon} = \hbar\omega = hc/\lambda$$

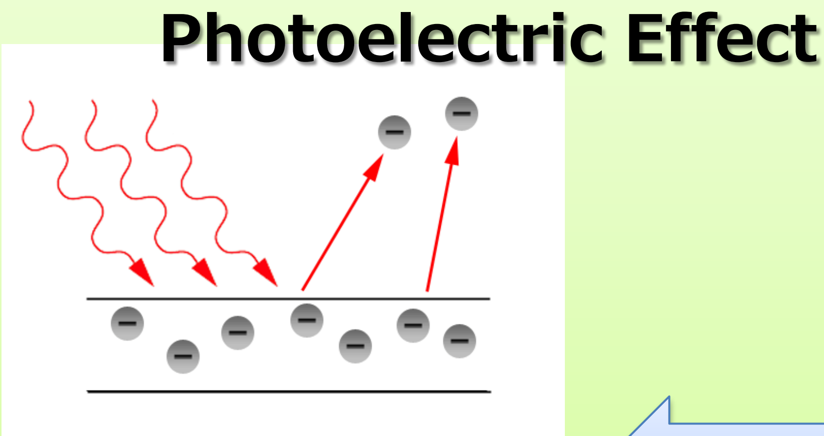
Photons with an energy of 12.4 keV



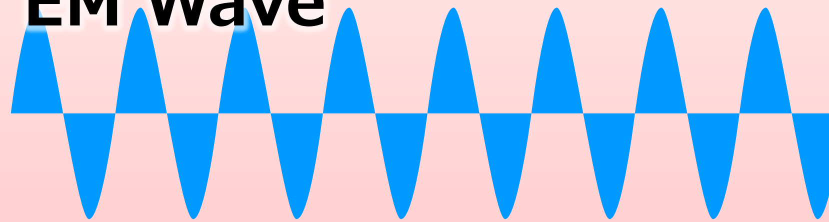
Wave? Photon?

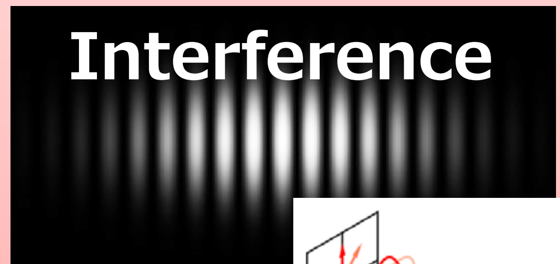
Photon





EM Wave





$$E_{photon} = hc/\lambda$$

✓Evaluation of SR characteristics (brilliance, flux, ⋯)

✓Formulation of SR with classical electrodynamics (field amplitude)

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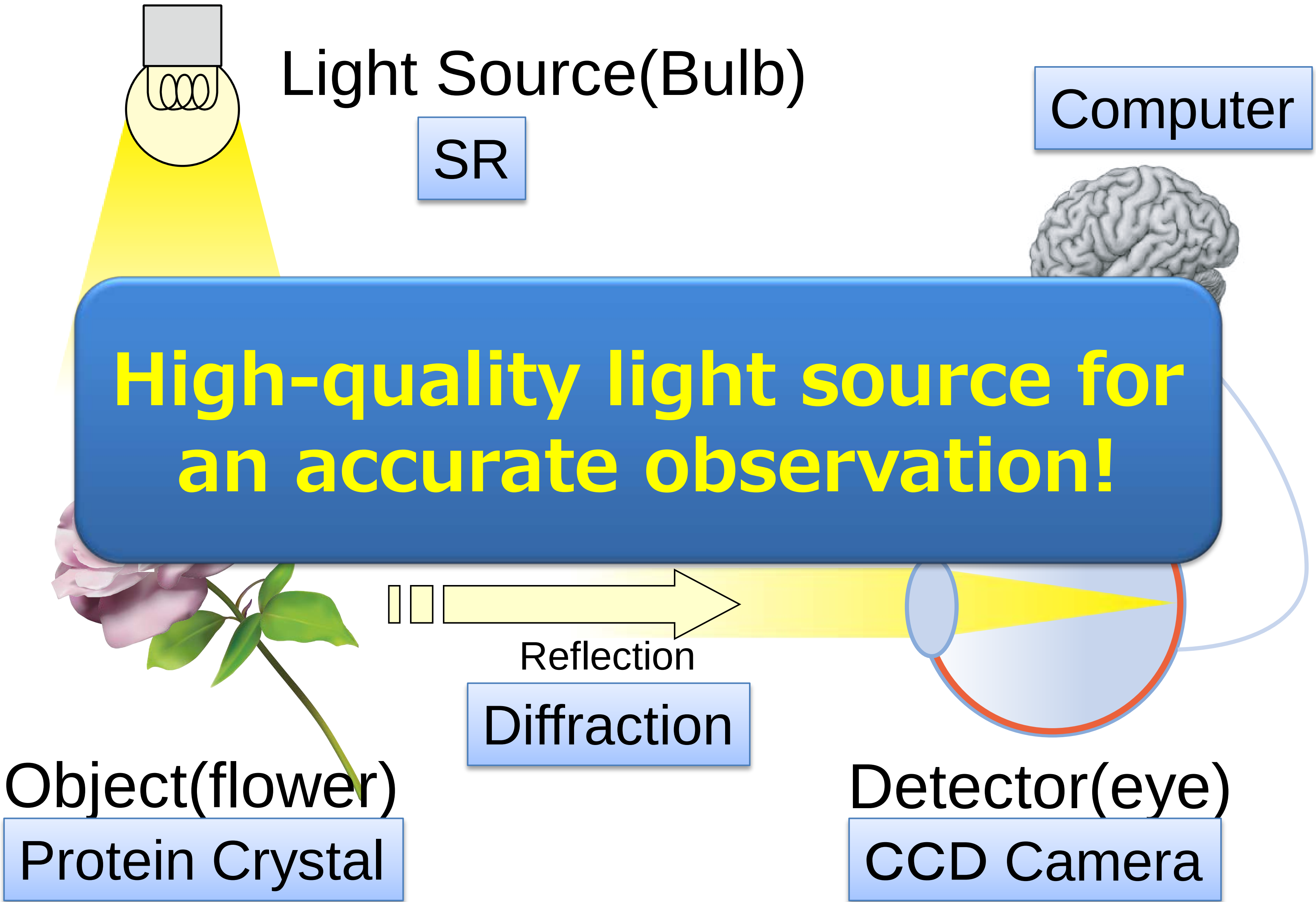
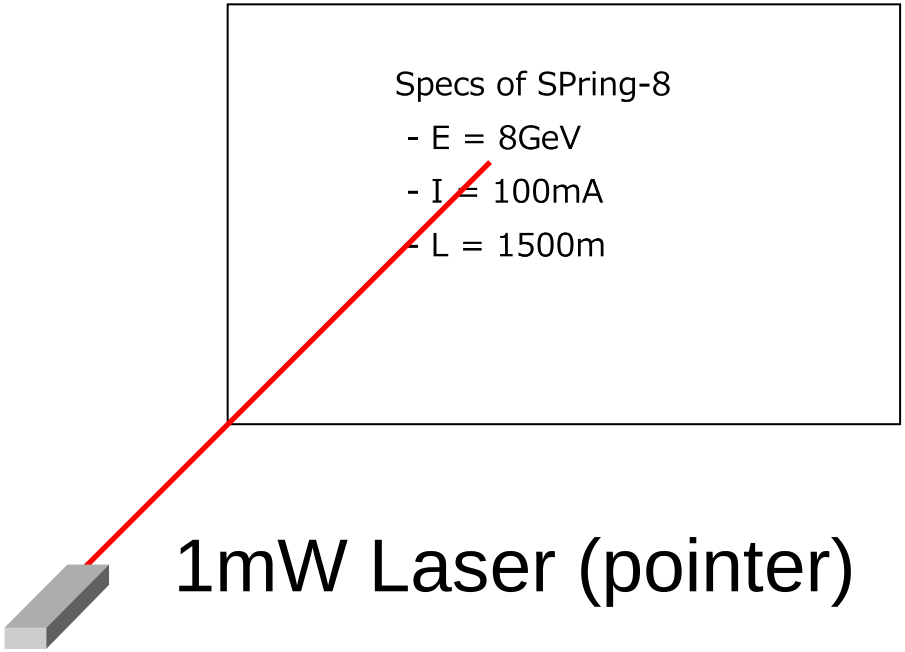
Observation with Light

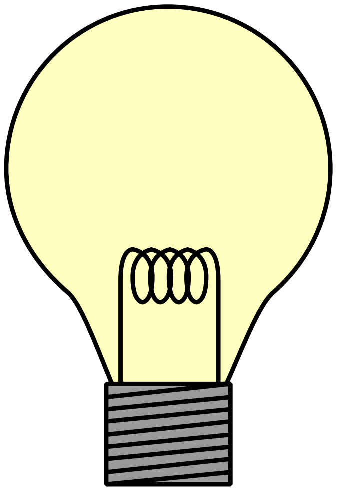


High-quality light source for an accurate observation!

Which Quality is Better?



1mW Laser (pointer)



100W Bulb

Lighting equipment in a room: **Bulb**

Pointer during a presentation: **Laser**

Depends on the Object!

How to Define the Quality of Light?(1)

How to Define the Quality of Light?(2)

Important Features of the Light Source

Item	Object		Why?
	Flower	Protein	
Radiation Power	◎	○	# Emitted Photons
Source Size	×	◎	Illuminated Area
Directivity	△	◎	
Monochromaticity	△	◎	Accuracy of Analysis

Brilliance

What is Brilliance ?

Brilliance(photons/sec/mm^2/mrad^2/0.1%B.W.)

$$\sim \frac{\text{Total Power}}{\text{Source Size x Angular Divergence x Band Width}}$$

- Brilliance specifies the quality of light for observation of microscopic objects.
- The brilliance of a light source with a high total power is not necessarily high.

Example of Brilliance Estimation

Item	Bulb	Laser Pointer
Total Power (W)	100	10^{-3}
Angular Div. ($mrad^2$)	$4\pi \times 10^6$	1
Source Size: (mm^2)	10^2	1
Bandwidth: (%)	100	0.01
Brilliance (photons/sec/....)	$\sim 10^8$	$\sim 10^{16}$

Laser is the best light source to observe the microscopic object!

X ray as a Probe

- Definition (not unique)
  - Electromagnetic wave (= light) with l of 10 nm(10-8 m) ~ 0.1 Å (10-11 m)
- Properties
  - High Energy/Photon
  - High Penetration (Roentgen etc..)
- Application to Microscopic Objects
  - X-ray Diffraction
  - Fluorescent X-ray Analysis
- No Practical Lasers!! (until recently)

➡ **Synchrotron Radiation (SR)**

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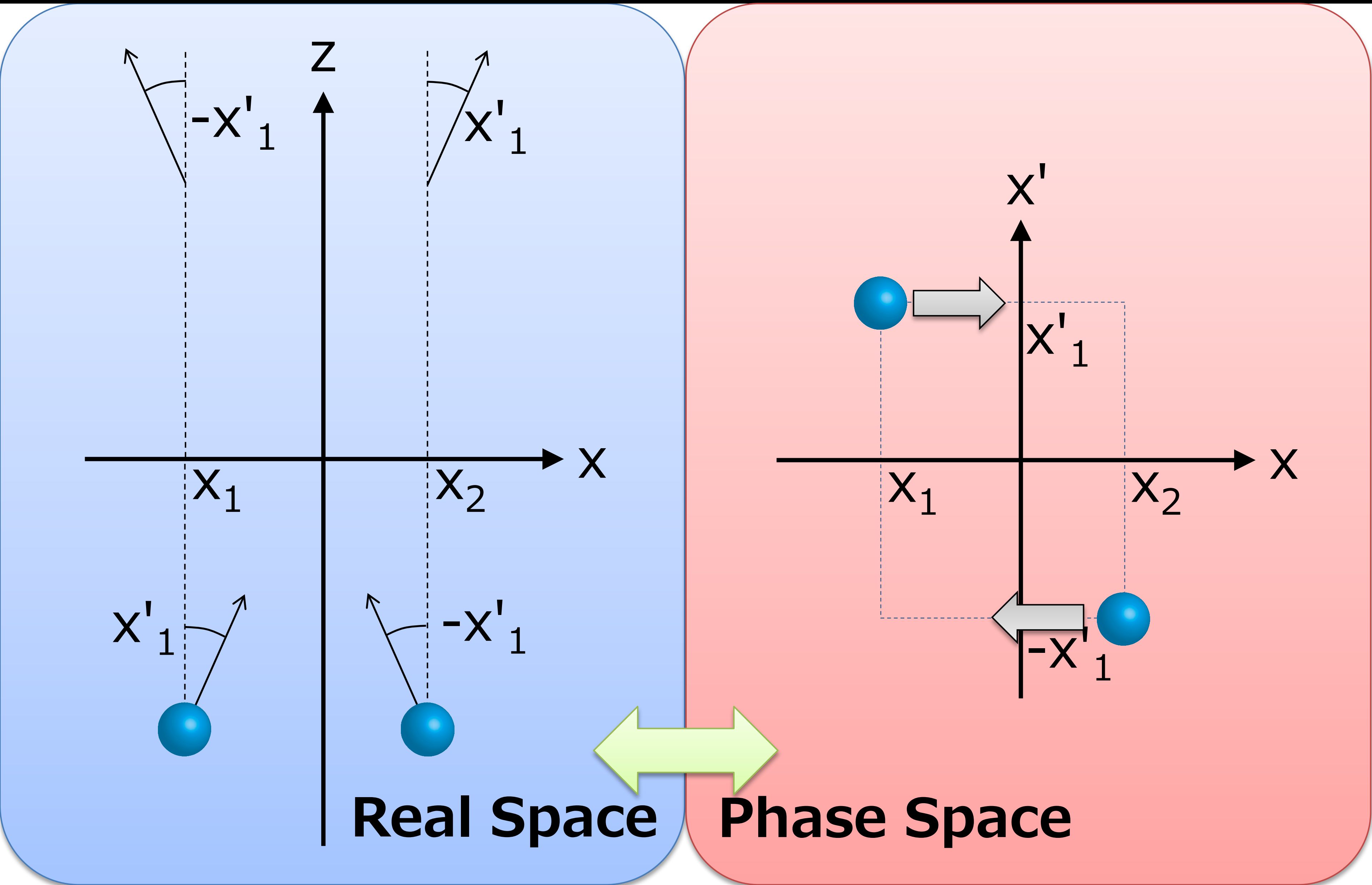
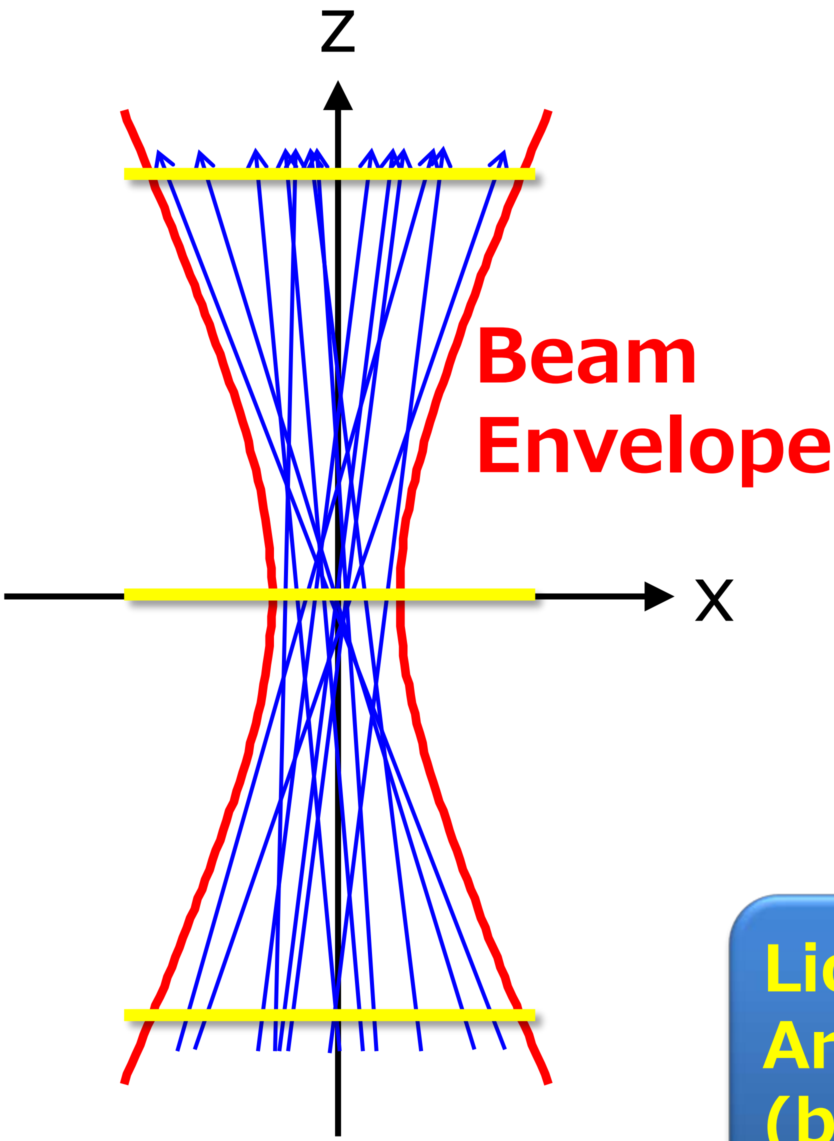
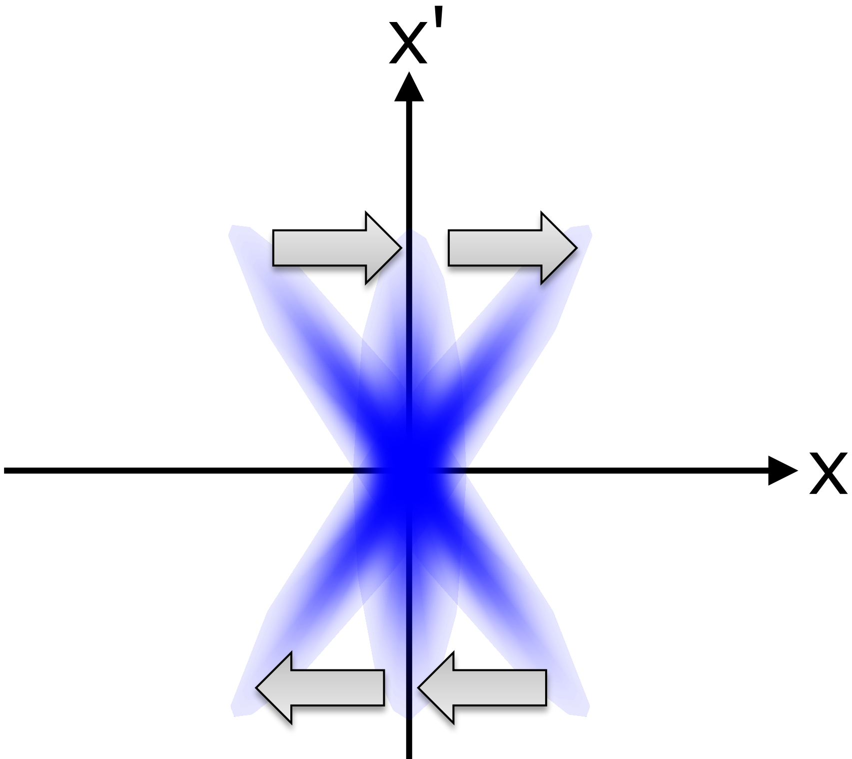
Phase Space Representation



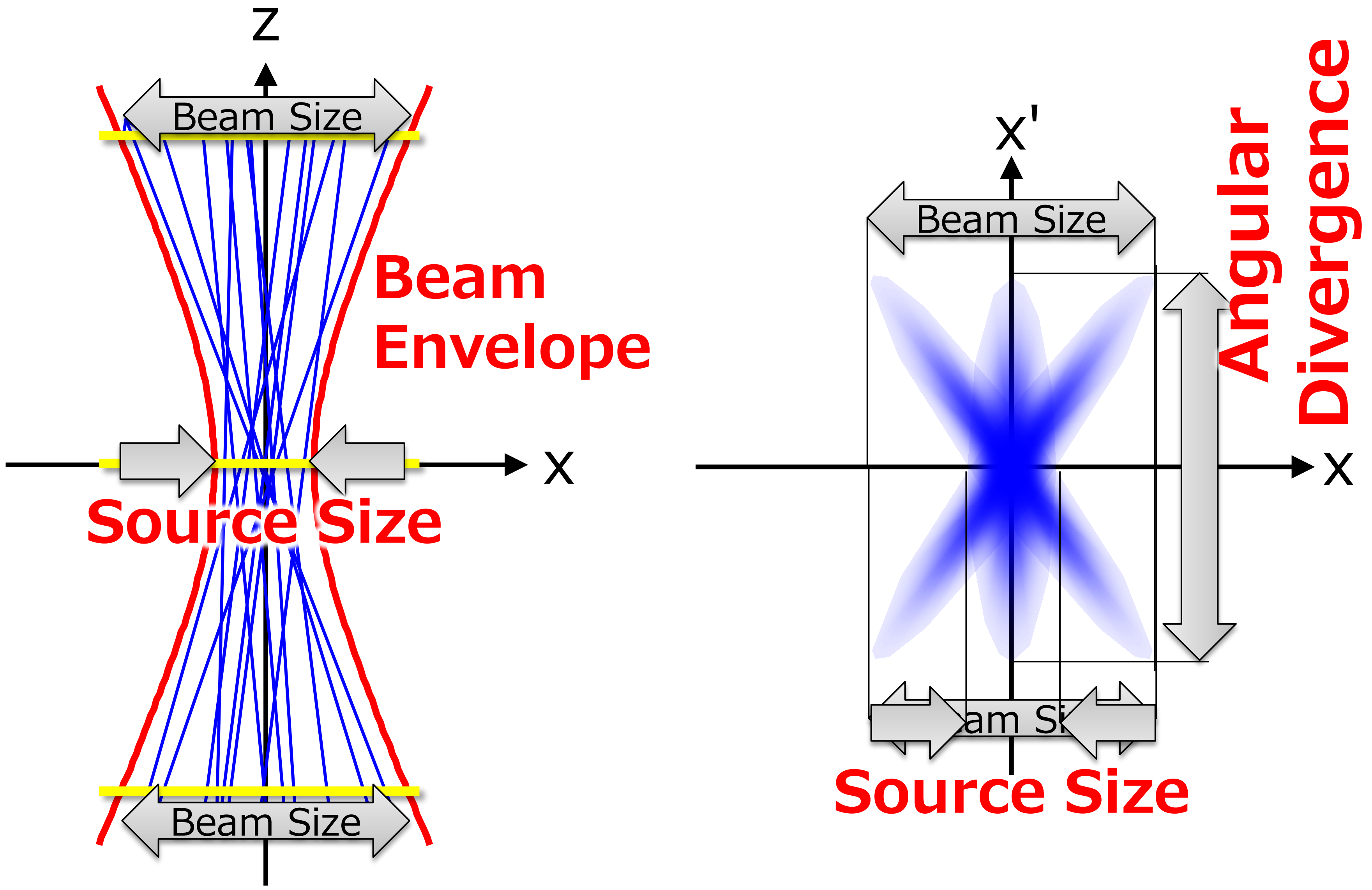
Photon Propagation in Phase Space



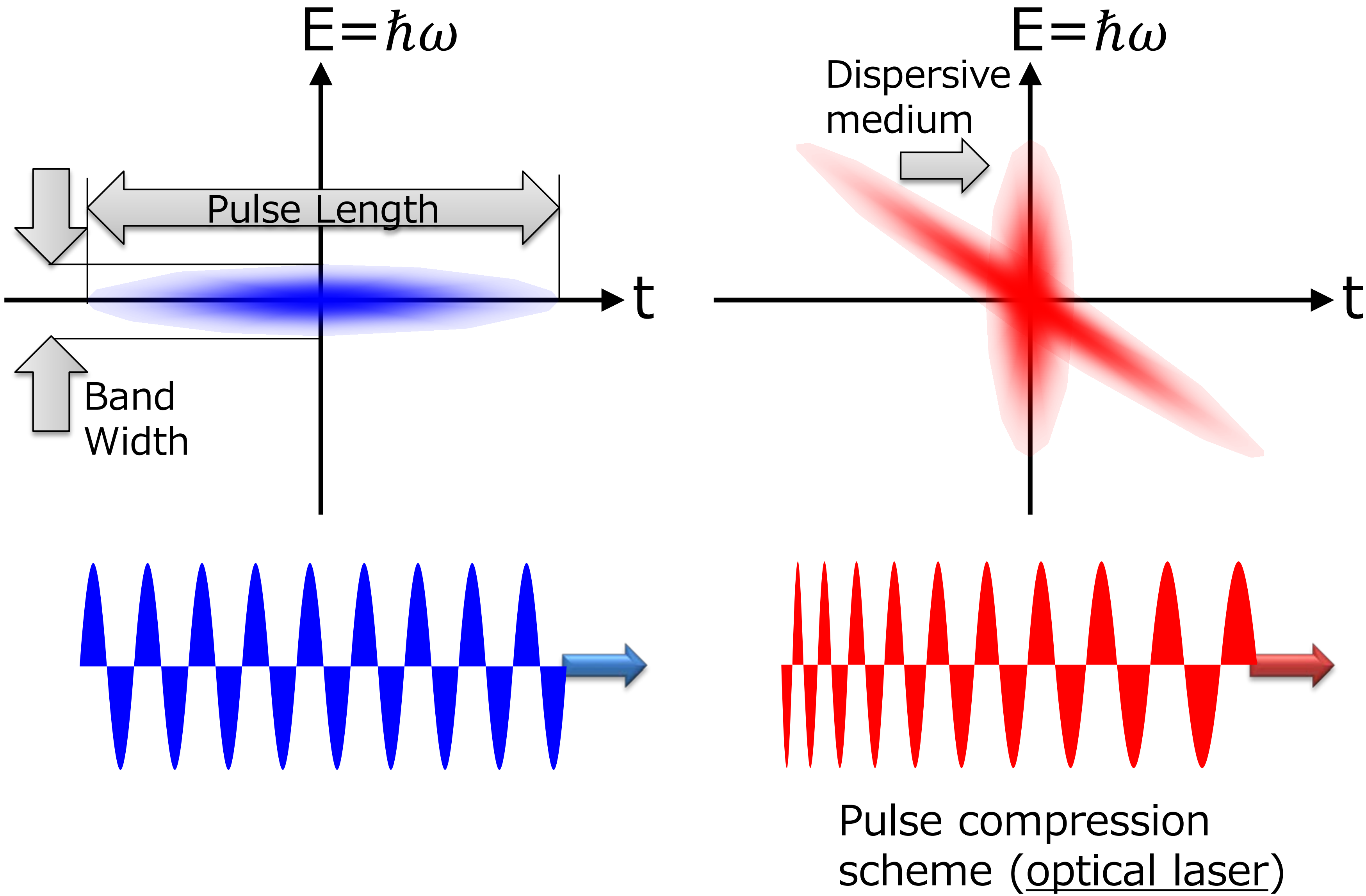


Liouville's Theorem:
Area of the phase ellipse (beam emittance) is constant.

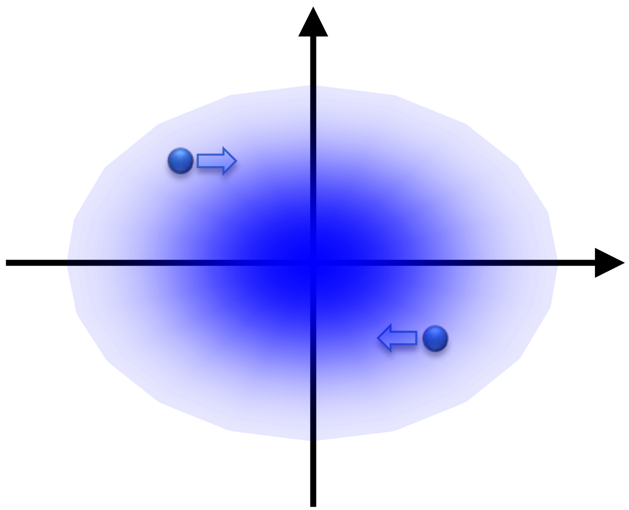
Photon Propagation in Phase Space



Energy-Time Phase Space



6D Phase Space & Brilliance



Motion of each photon can be described in a space spanned by the 6 coordinates (phase space)

Photon distribution function in the 6D phase space gives full information on light source

Typical form of the photon distribution

Central Freq.

$$n(x, x', y, y', t, \omega) = B_0 \exp\left[-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{x'^2}{2\sigma_{x'}^2} - \frac{y'^2}{2\sigma_{y'}^2} - \frac{t^2}{2\sigma_t^2} - \frac{(\omega - \omega_c)^2}{2\sigma_\omega^2}\right]$$

Brilliance (B_0)

Source Size (σ_x, σ_y)

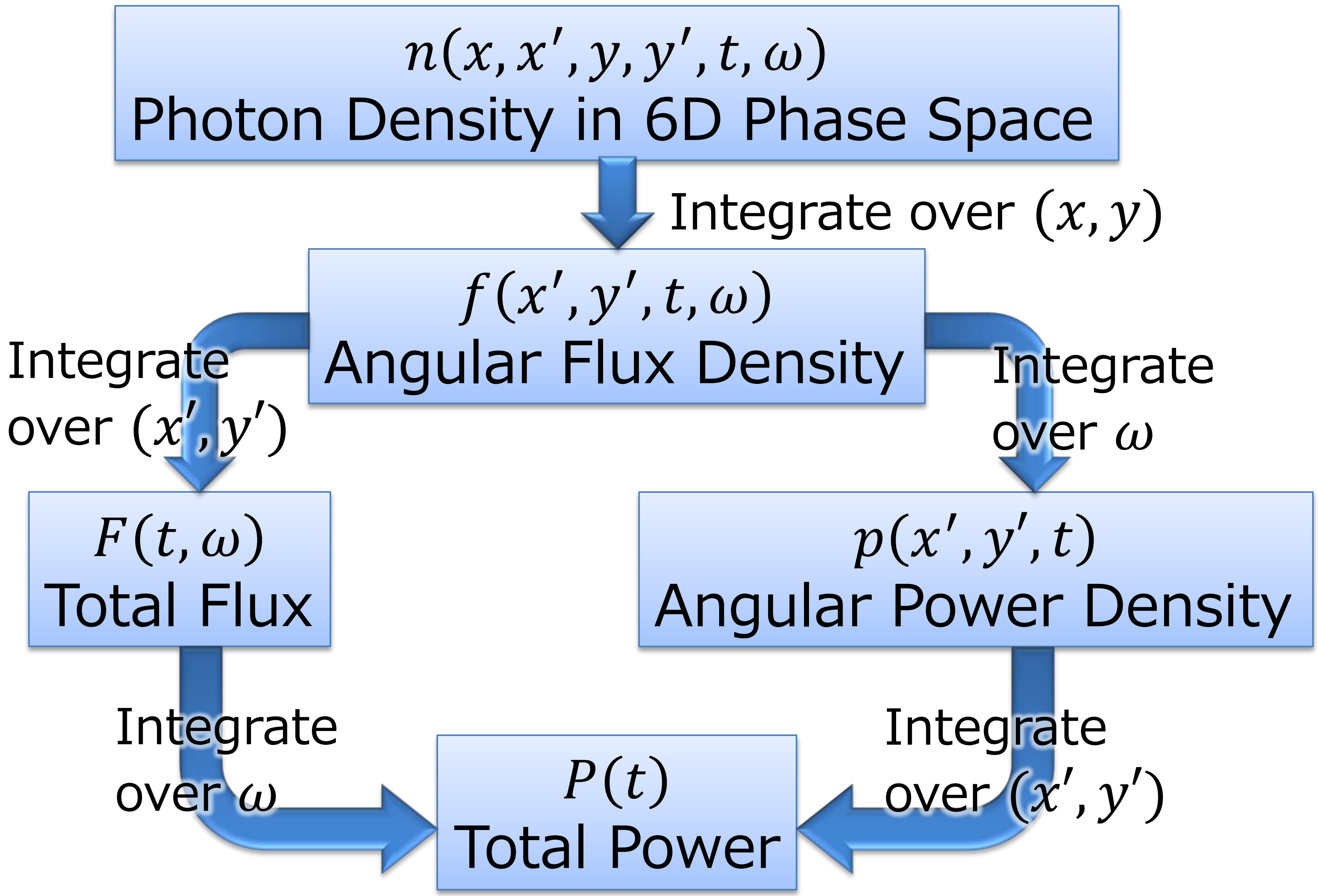
Angular Divergence ($\sigma_{x'}$, $\sigma_{y'}$)

Pulse Length (σ_t)

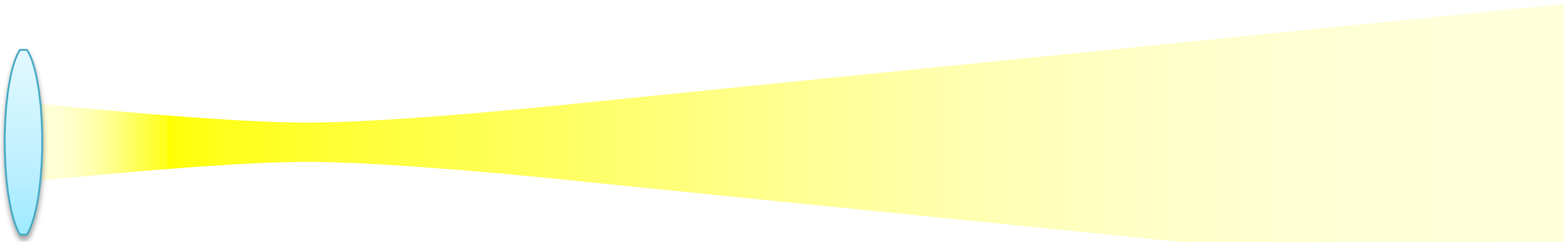
Band Width (σ_ω)

Central Freq. (ω_c)

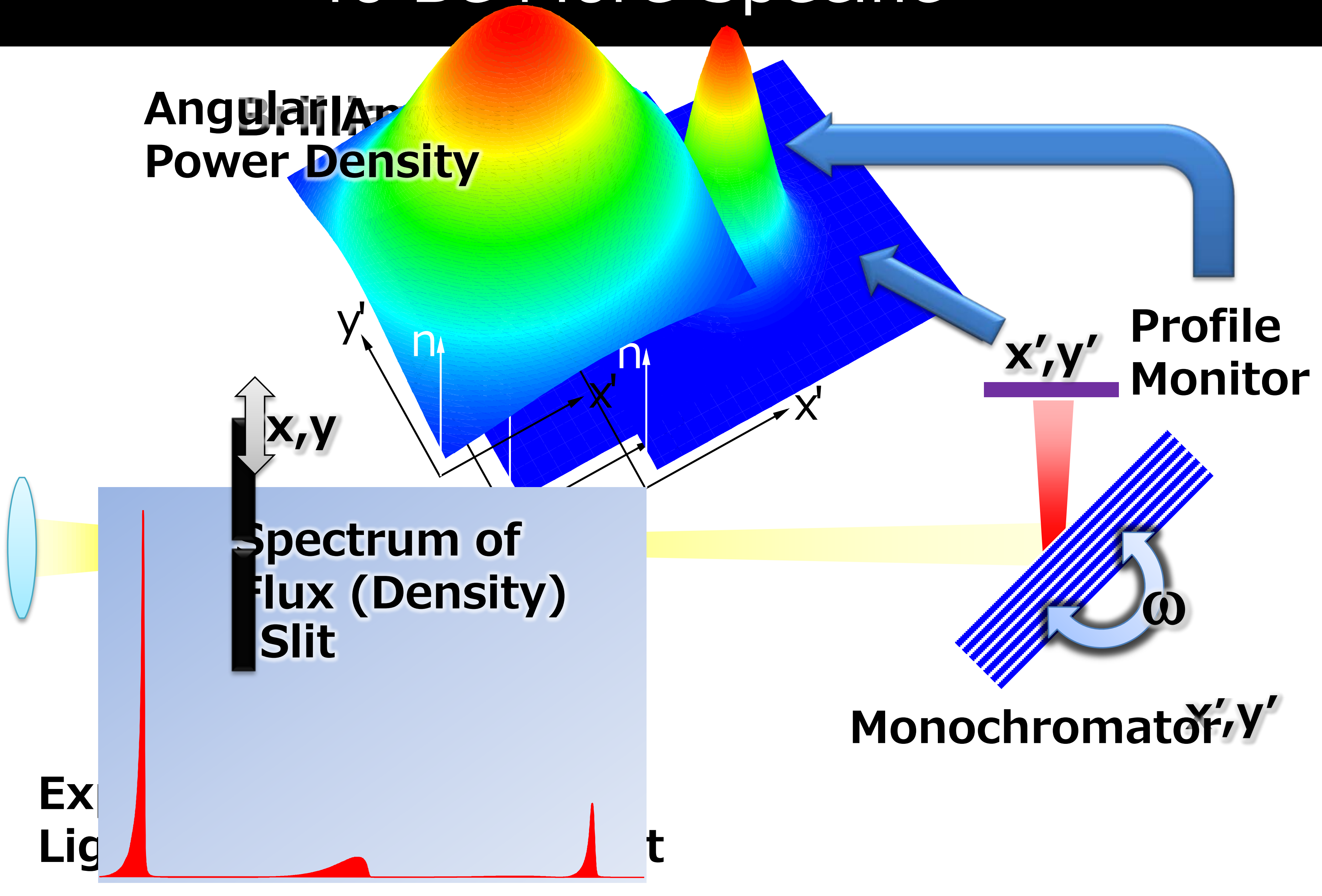
Photon Flux & Radiation Power



To Be More Specific···



To Be More Specific…



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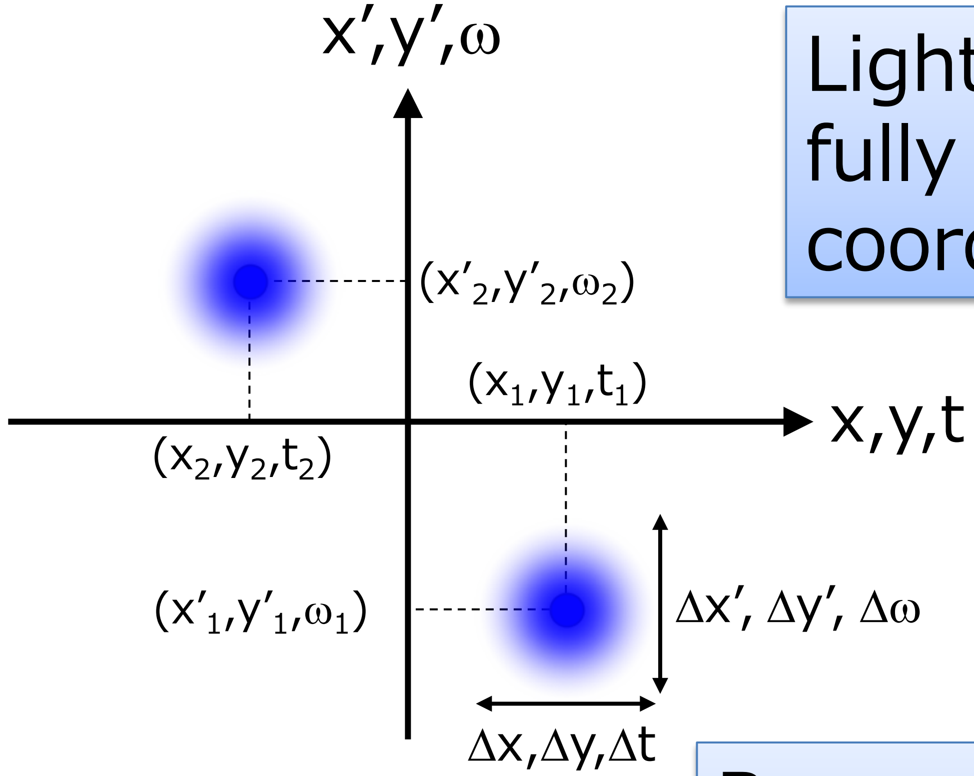
Uncertainty of Light

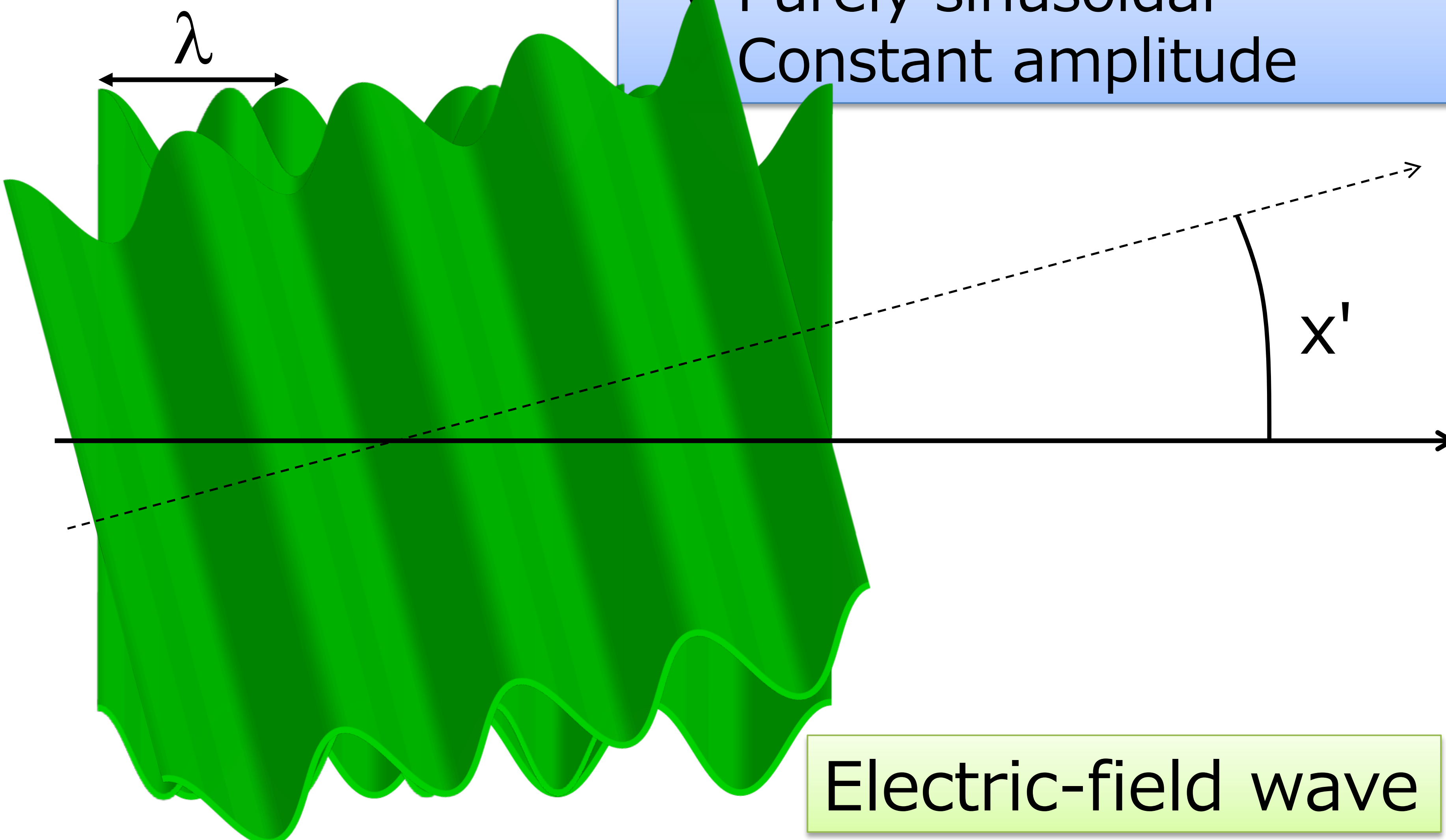


Light as a photon can be fully described by the 6 coordinate variables.

Because of the **wave nature**, they have some uncertainty characterized by the so-called Fourier transform.

Monochromatic & Plane Wave

Monochromatic & Plane Wave
- ✓ Purely sinusoidal
- ✓ Constant amplitude

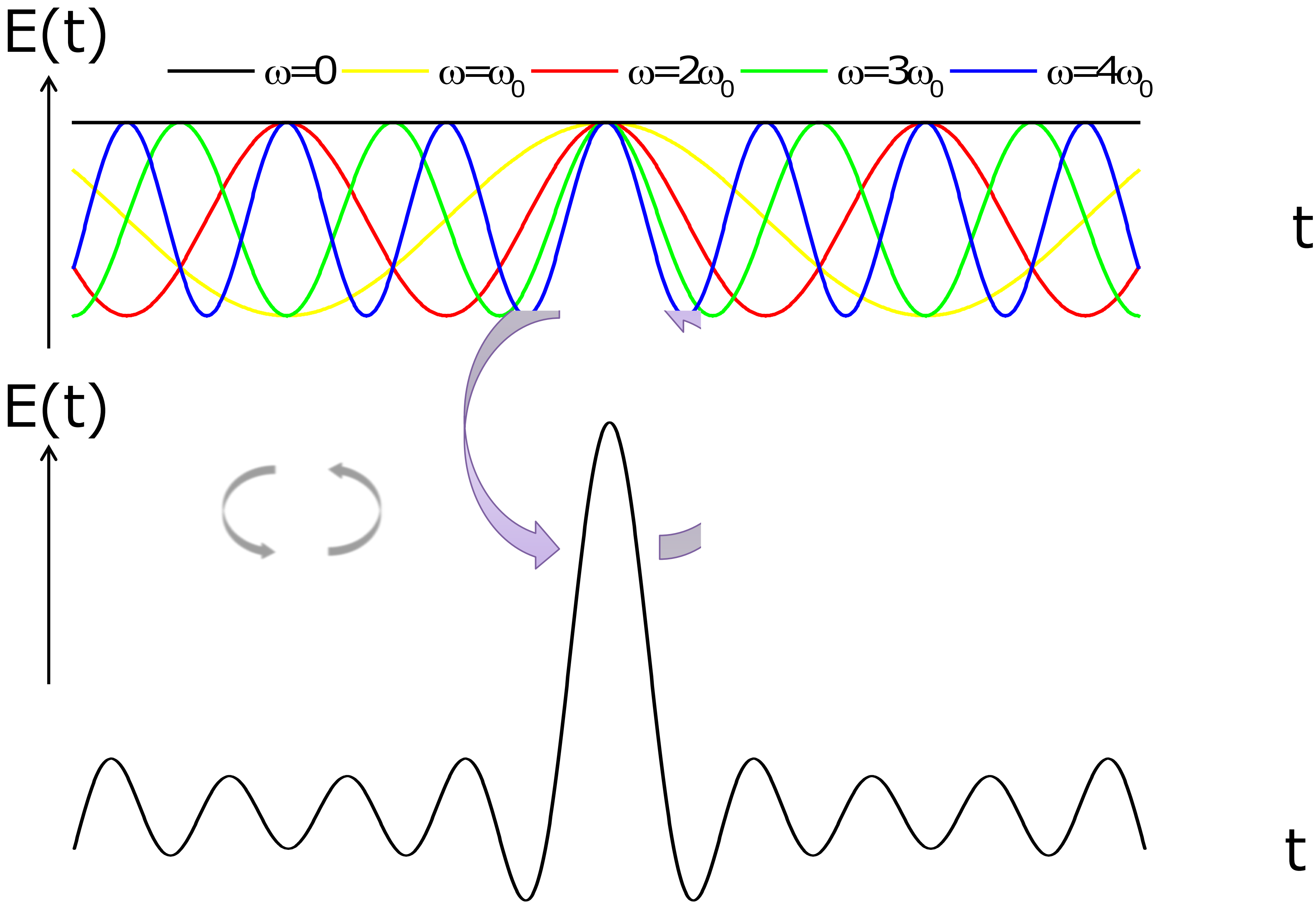


Electric-field wave

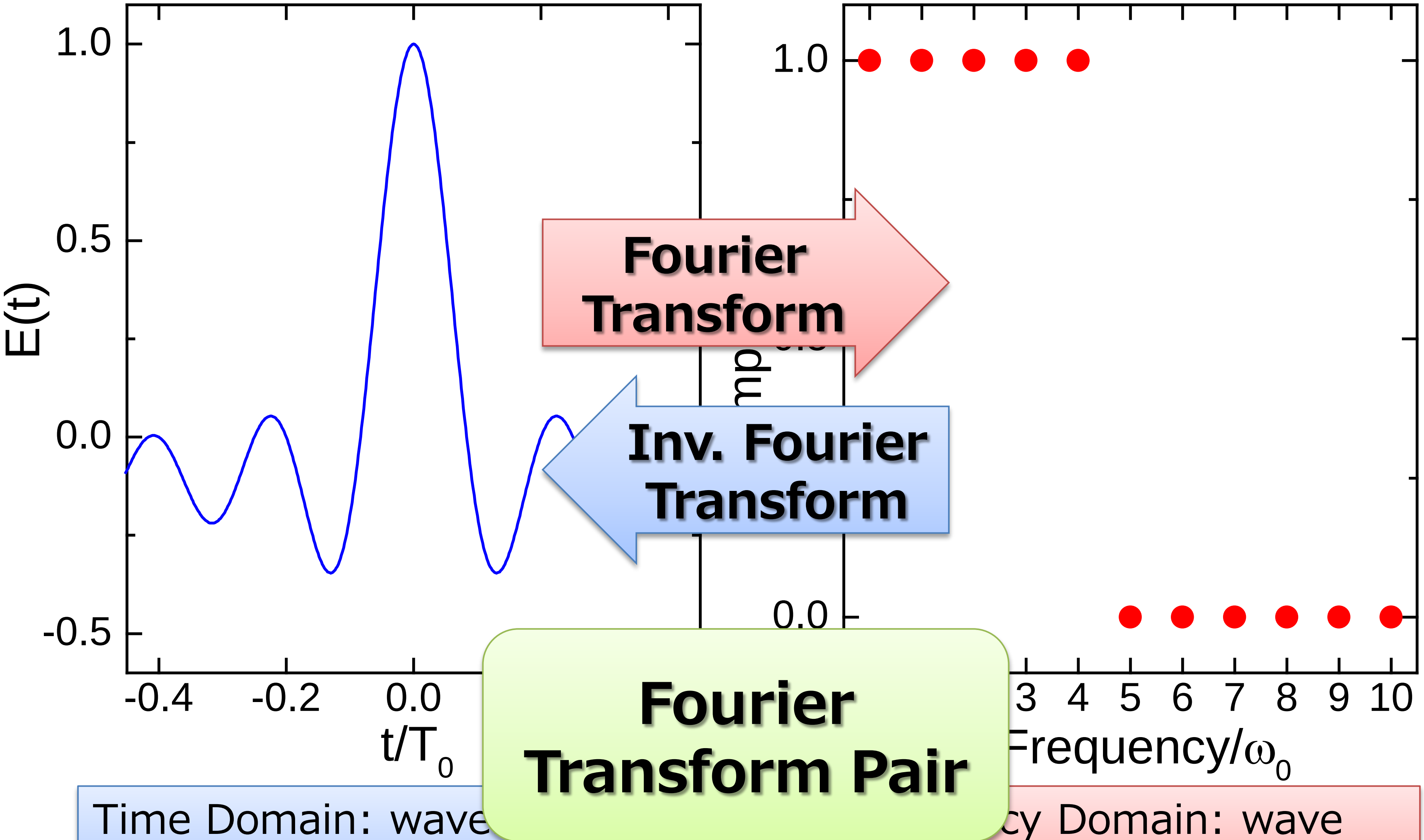
In Reality…

- A monochromatic and plane wave is an (ideal) form of a wave.
- In practice, a wave is composed of many ideal waves having different λ and x'.
- **Fourier Transform** is a mathematical operation to decompose an arbitrary wave into a number of monochromatic and plane waves.

Composition of Monochromatic Waves



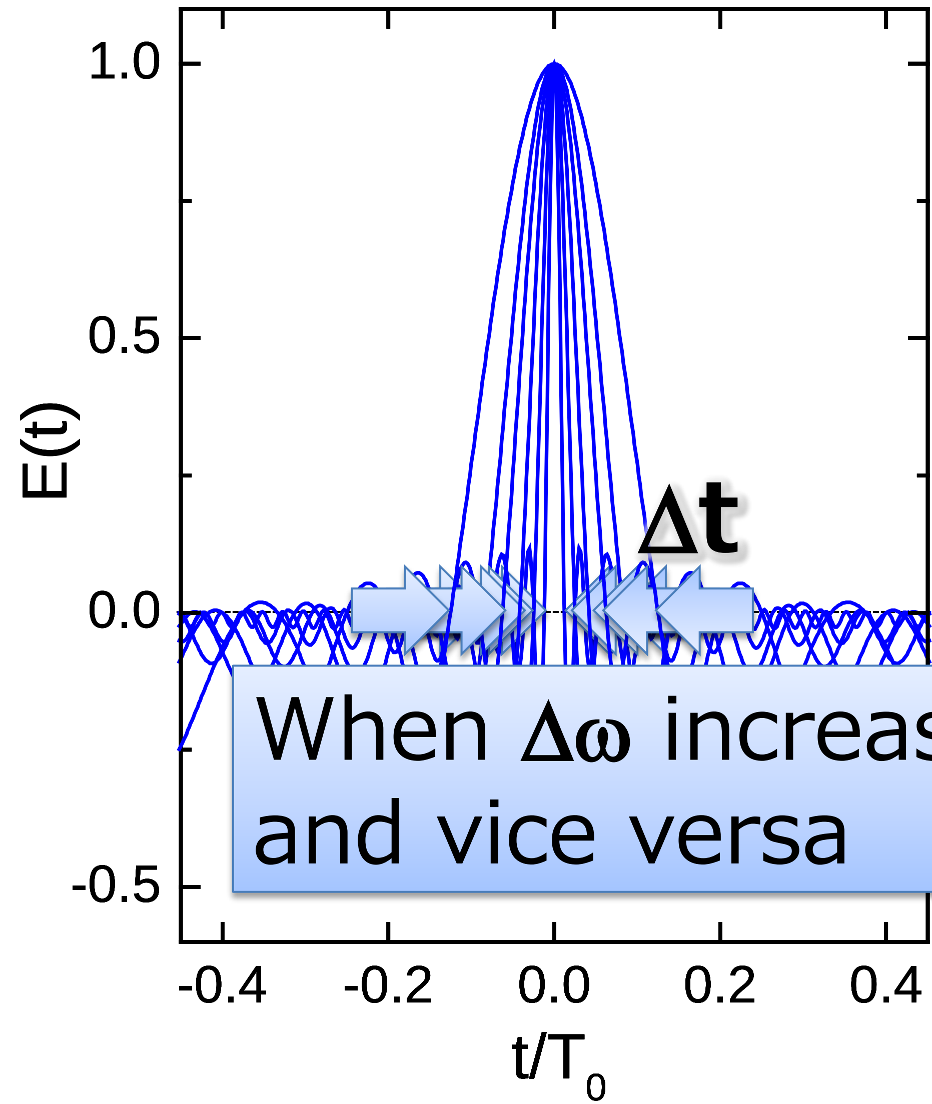
Time & Frequency Domains

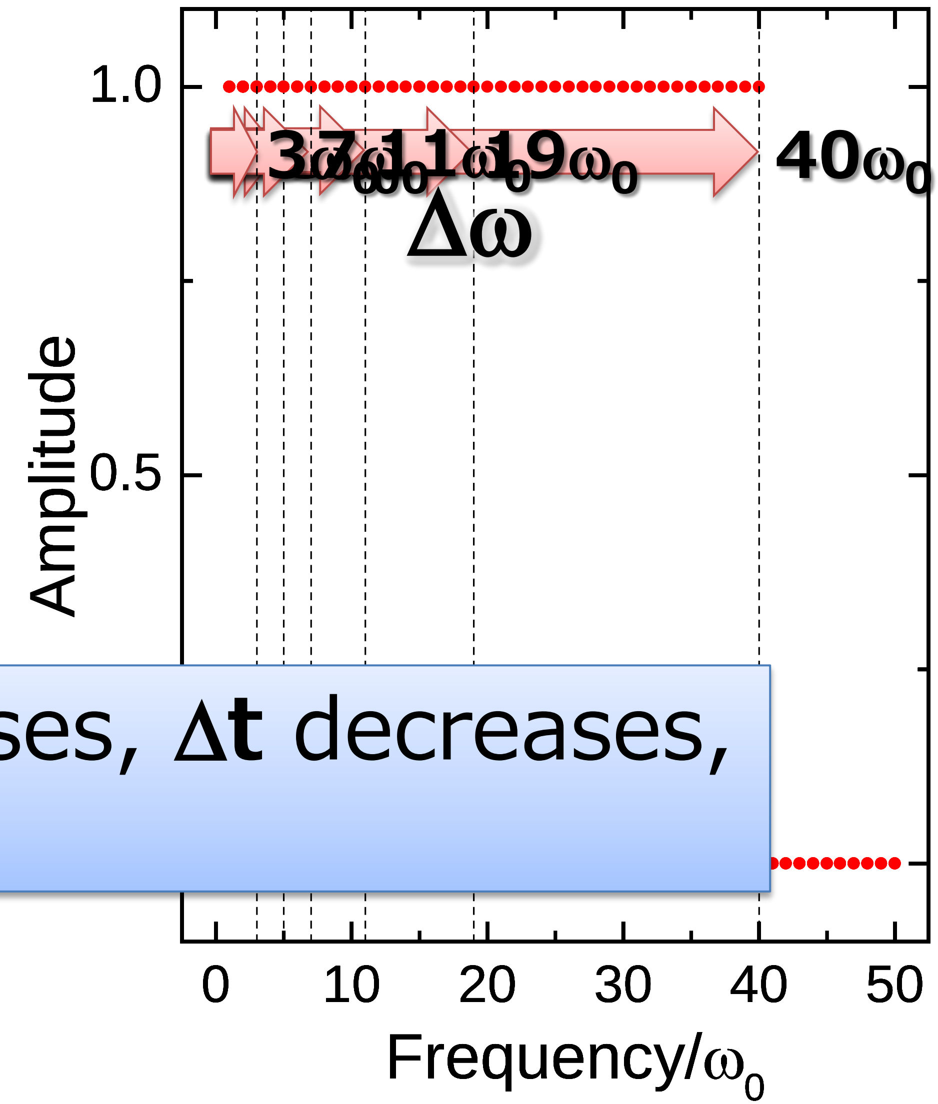


Time Domain: wave … a function of t

…cy Domain: wave amplitude as a function of ω

How Does the Waveform Changes?

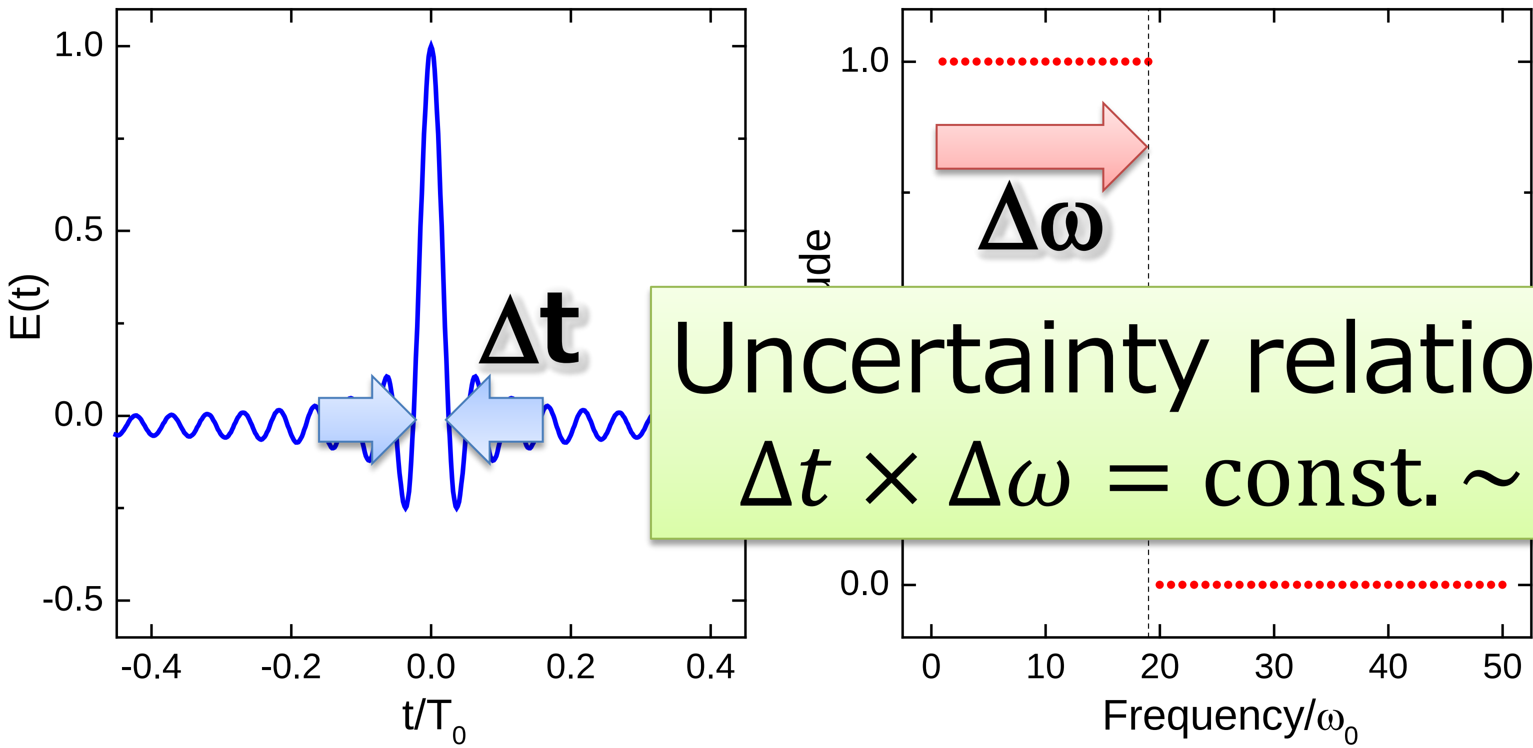




When Δω increases, Δt decreases, and vice versa

Uncertainty Relation

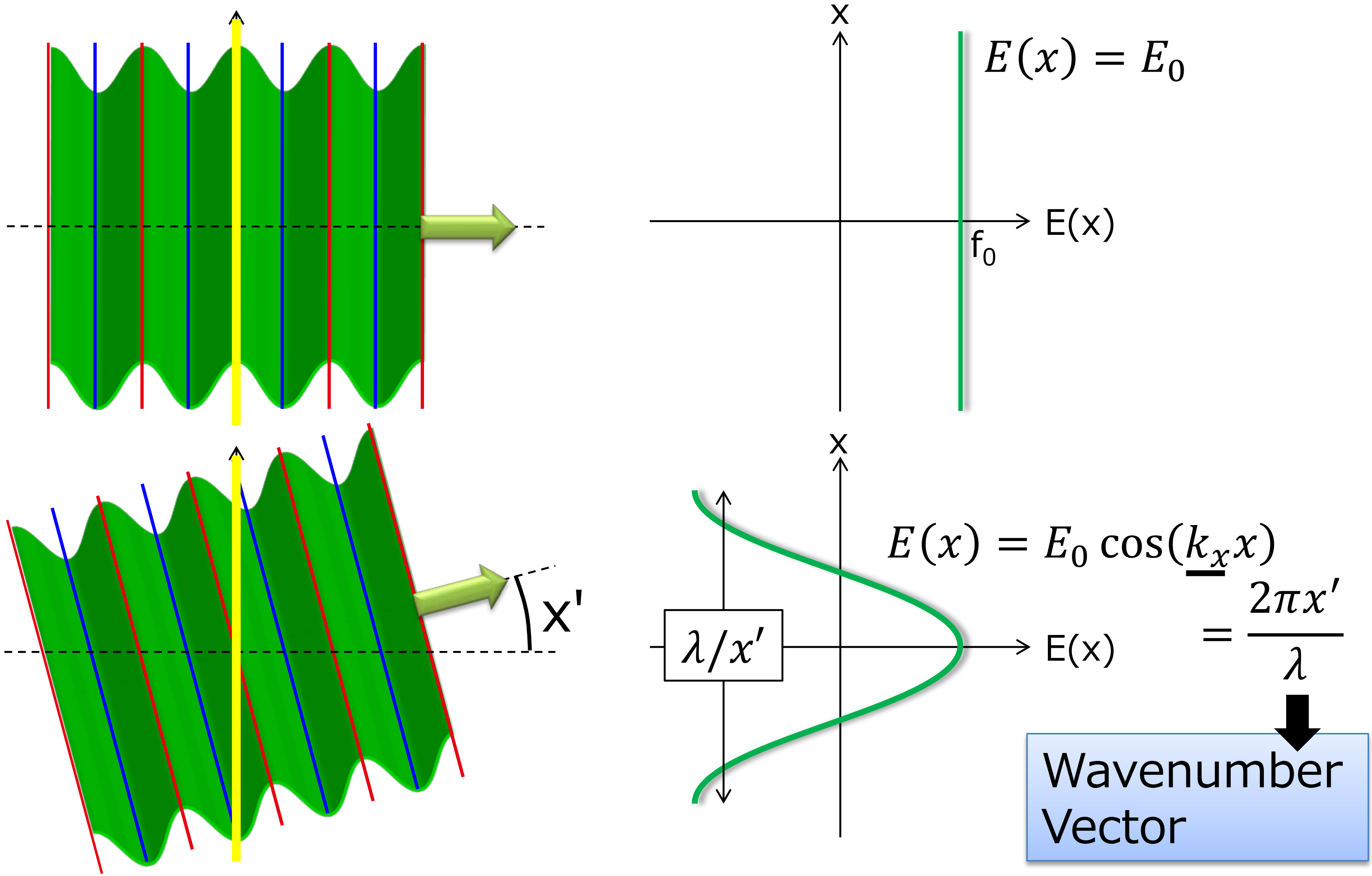
- **Δt** and **Δω** are regarded as uncertainty of a photon
  - **Δt:** longitudinal position (pulse width)
  - **Δω:** photon energy (bandwidth)



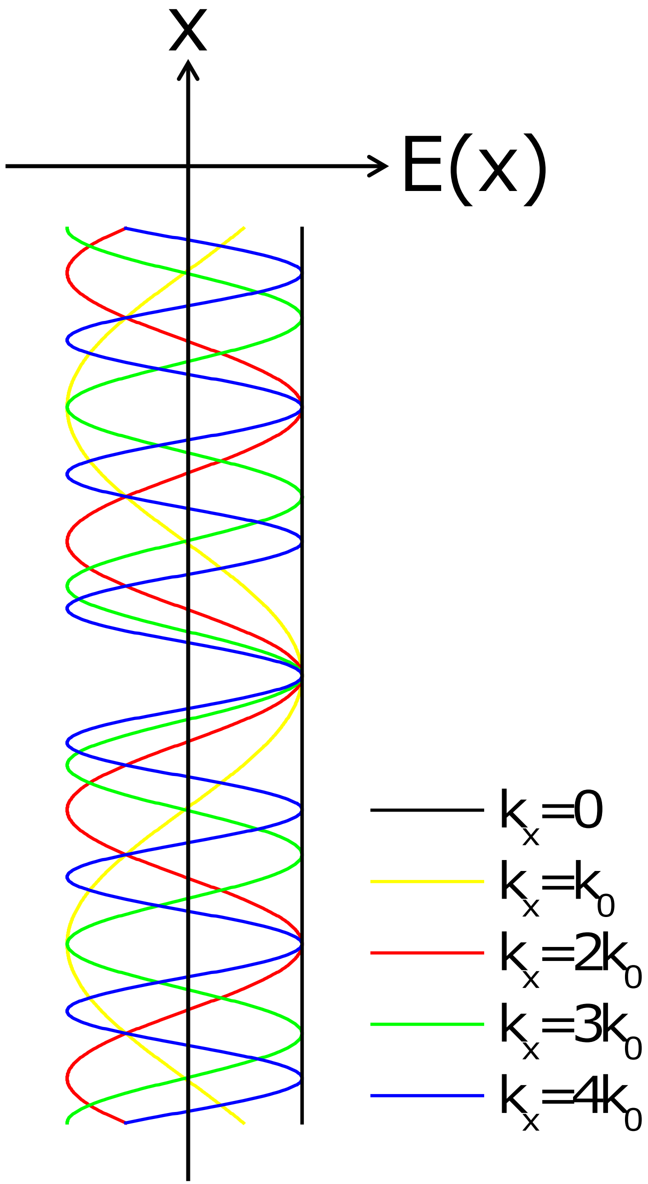
Uncertainty relation:

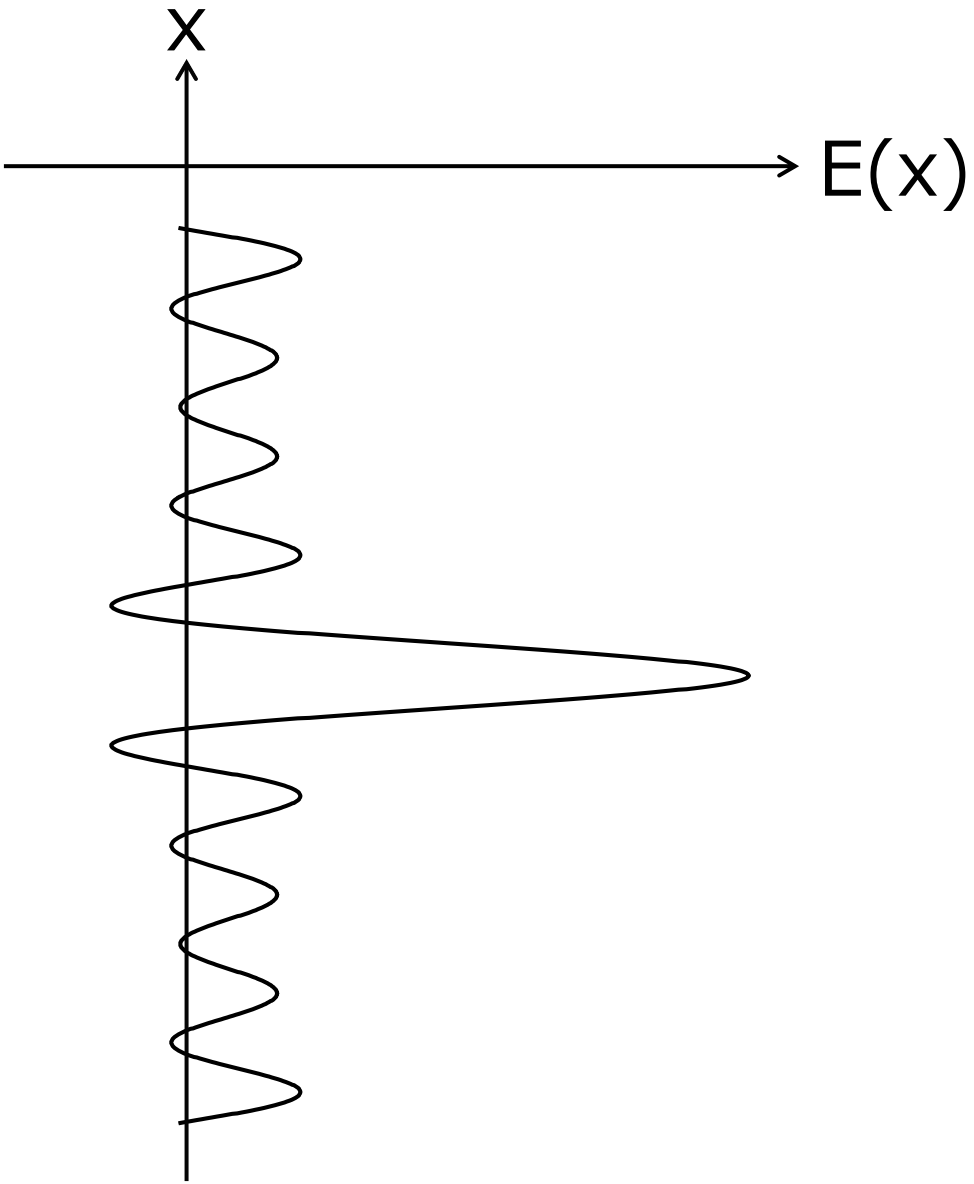
$$\Delta t \times \Delta\omega = \text{const.} \sim 1$$

Composition of Plane Waves

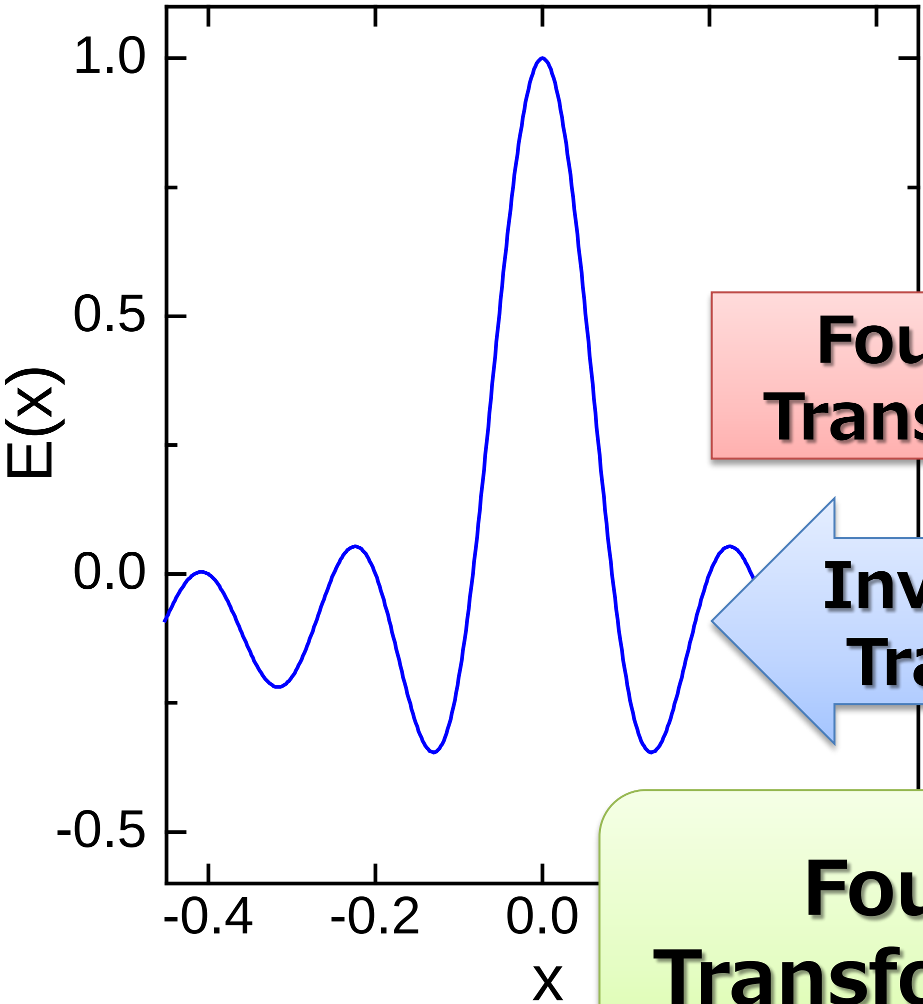


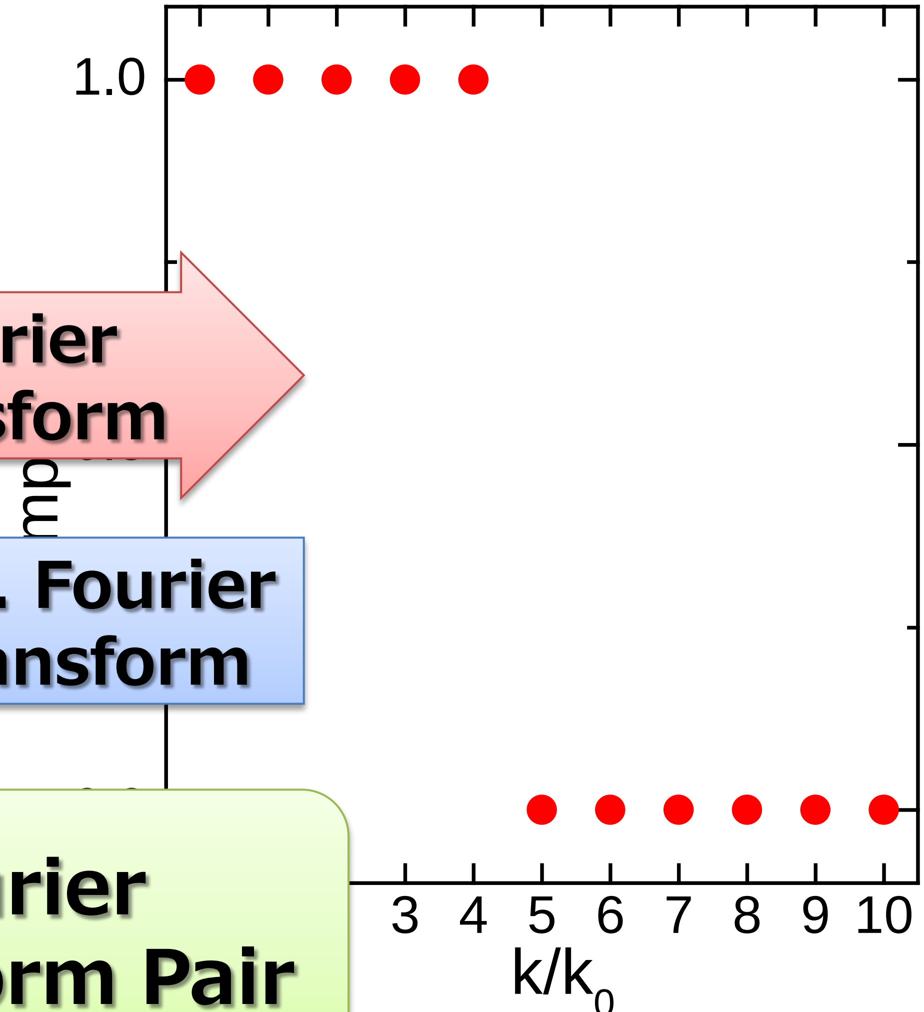
Composition of Plane Waves





Spatial & Angular Domains





Fourier Transform

Inv. Fourier Transform

Fourier Transform Pair

Spatial Domain: waveform as a function of x

Angular Domain: wave amplitude as a function of k

Fourier Transform Examples

$F(t)$ **Fourier Transform** $\tilde{F}(\omega) = \int F(t) e^{i\omega t} dt$

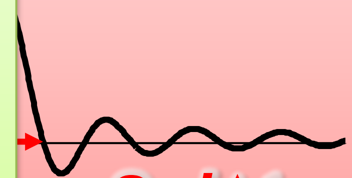
Boxcar

$\begin{cases} 1/\Delta t \\ 0 \end{cases}$

Product of "widths" of Fourier transform pair (F and $\tilde{F}$) is a constant

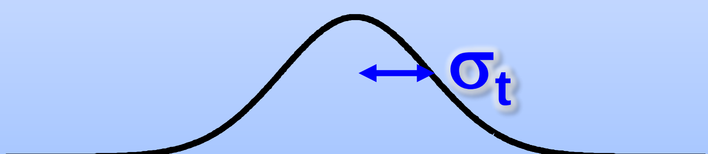
c" Function

$\mathrm{sinc}(\omega\Delta t/2)$



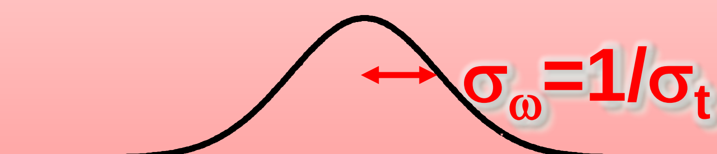
Gaussian Function

$$\frac{1}{\sqrt{2\pi}\sigma_t} \exp\left(-\frac{t^2}{2\sigma_t^{\,2}}\right)$$

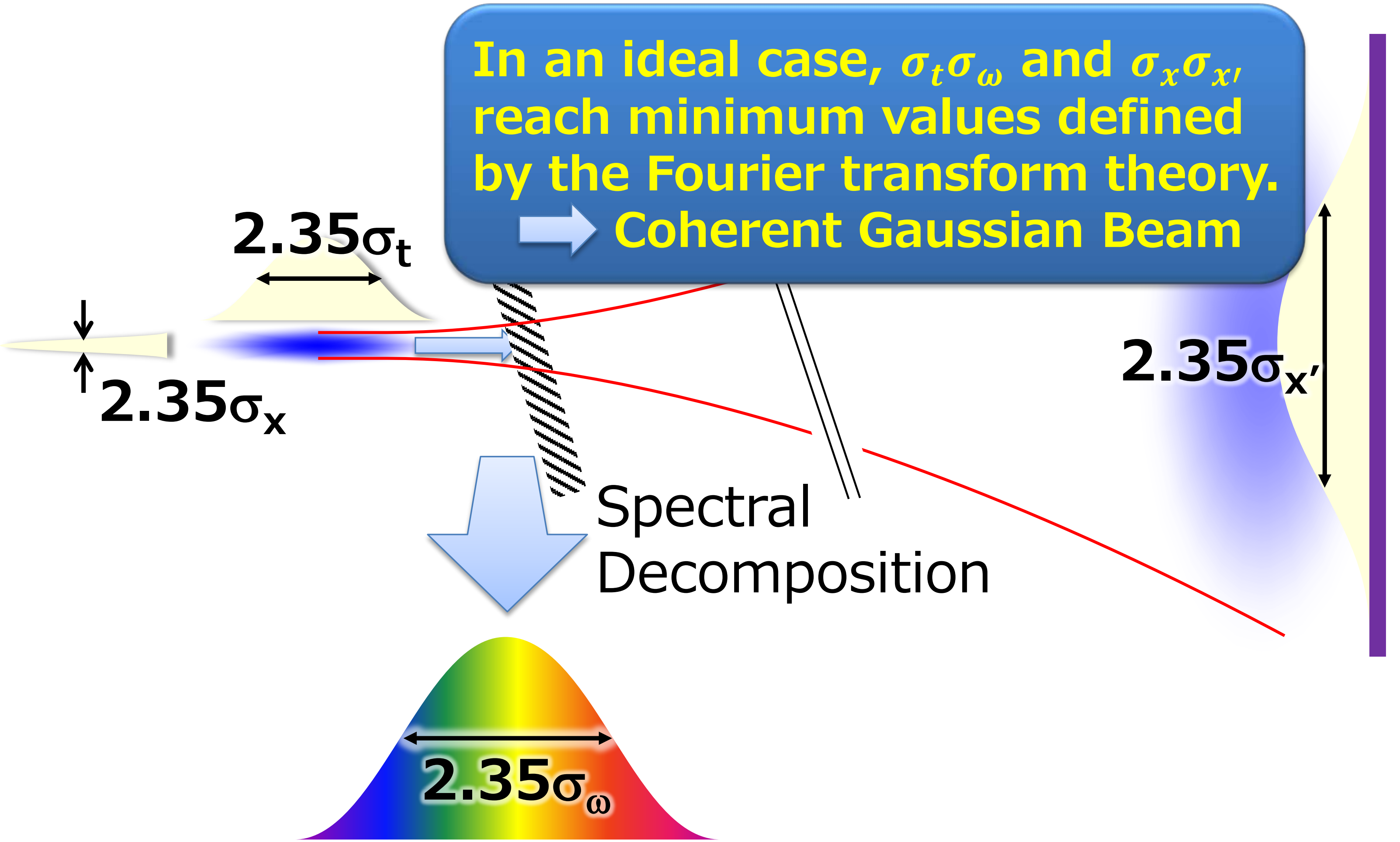


Gaussian Function

$$\exp\left(-\frac{\omega^2 \sigma_t^{\,2}}{2}\right)$$



Coherent Gaussian Beam



Fourier and Diffraction Limits

	Condition	Extreme Cases	
Fourier Limit	a) temporal $\sigma_t\sigma_\omega \geq \frac{1}{2}$	$\sigma_t = 0$ $\sigma_\omega = \infty$	White (or Pulse) Light
		$\sigma_t = \infty$ $\sigma_\omega = 0$	Monochromatic Light
Diffraction Limit	b) spatial $\sigma_x\sigma_{x'} \geq \frac{\lambda}{4\pi}$ $\sigma_y\sigma_{y'} \geq \frac{\lambda}{4\pi}$	$\sigma_x = \infty$ $\sigma_{x'} = 0$	Parallel Light
		$\sigma_x \sim \lambda$ $\sigma_{x'} \sim 1$	Minimum Focal Size

- If equality a) holds, the light is:
  - **Temporally Coherent** or **Fourier Limited**
- If equality b) holds, the light is:
  - **Spatially Coherent** or **Diffraction Limited**

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SR: Light from a Moving Electron

- Unlike the ordinary light source (sun, light bulb,...), the light emitter of SR (electron) is ultra-relativistic.
- The characteristics of SR is thus quite different because of relativistic effects.
- What we have to take care is:
  1. Speed-of-light limit
  2. Squeezing of light pulse
  3. Conversion of the emission angles

Speed-of-Light Limit

Within the framework of relativity, the velocity of any object never exceeds the speed of light.

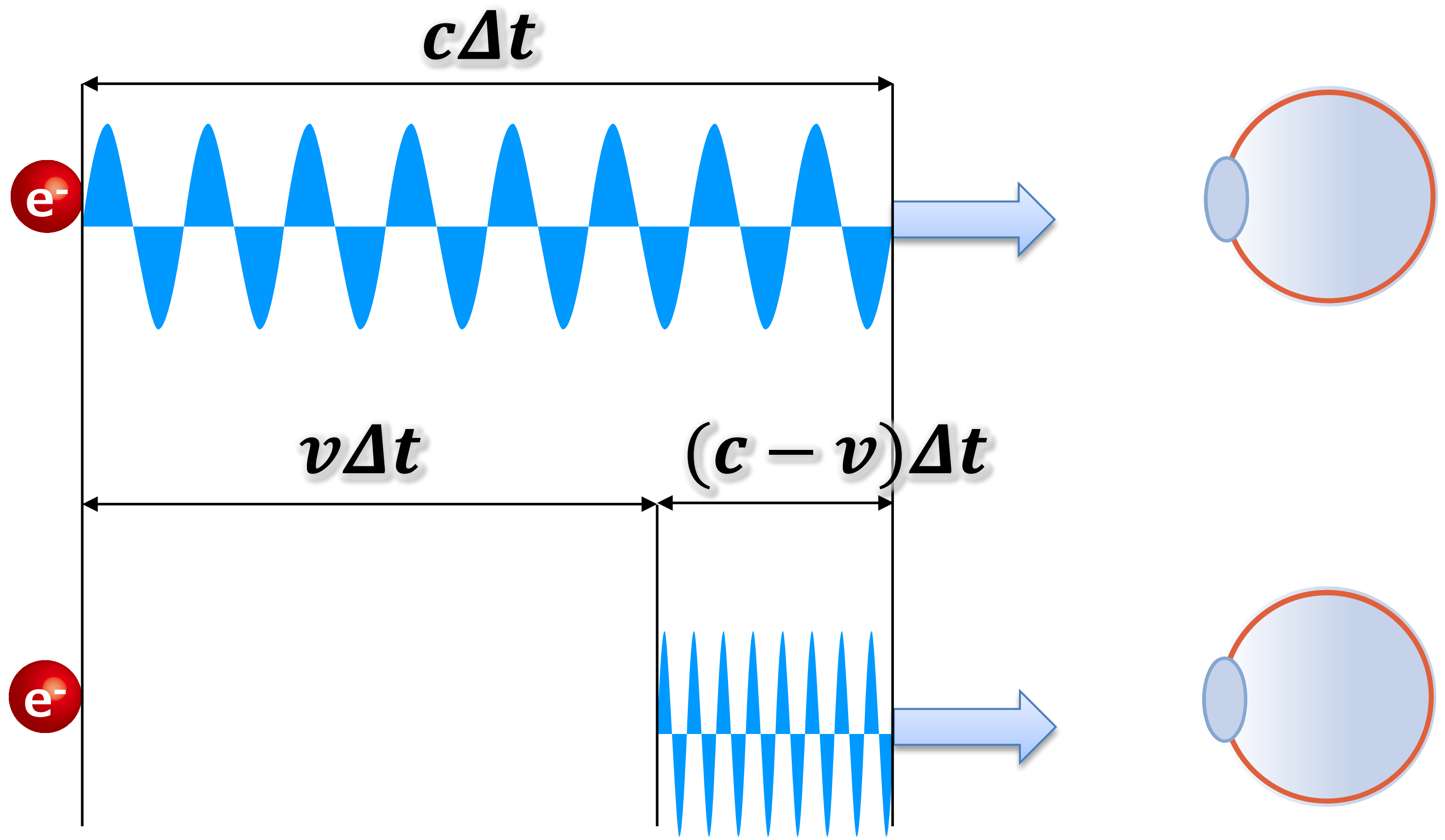
$$v/c = \beta = \sqrt{1-\gamma^{-2}}$$
$$\sim 1 - \frac{1}{2\gamma^2}$$

Energy	β
1MeV	0.941
10MeV	0.9988
100MeV	0.999987
8GeV	0.999999998

$$\gamma = \frac{E}{mc^2}$$

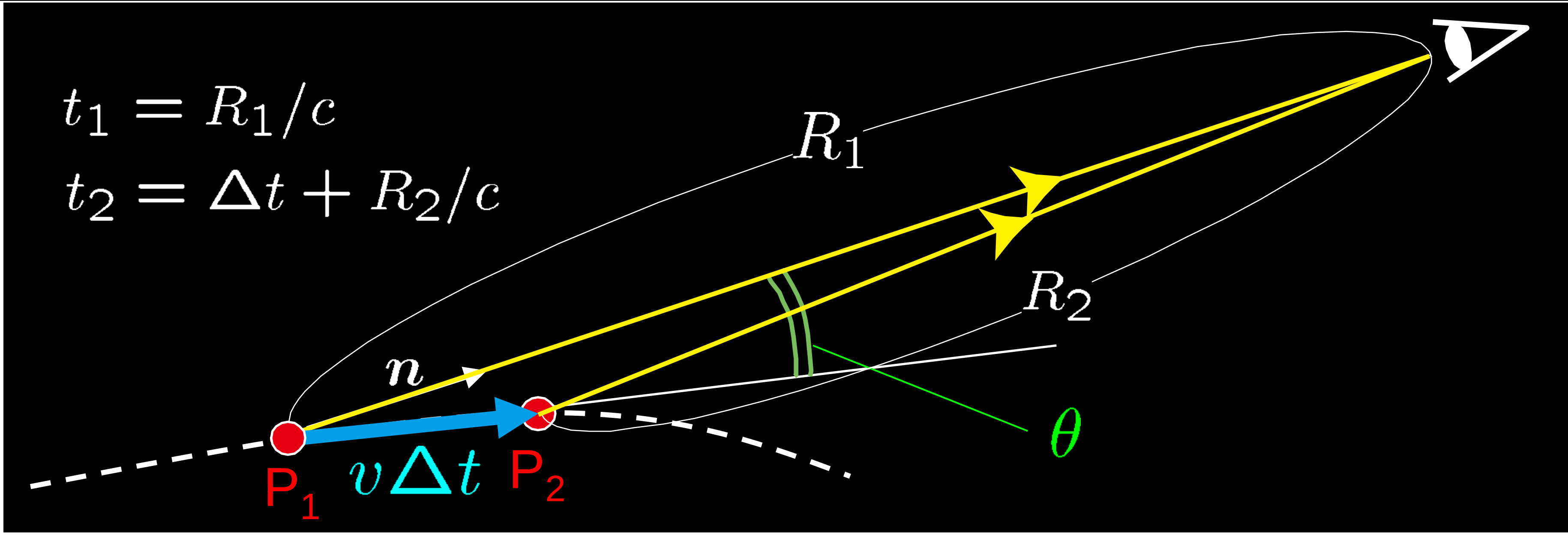
:Lorentz Factor
(relative electron energy, mc^2=0.511MeV)

Squeezing of Light Pulse Duration



$$\text{Pulse Duration Ratio} = \frac{(c-v)\Delta t}{c\Delta t} = 1 - \beta \sim \frac{1}{2\gamma^2}$$

More Generally⋯



$$R_2 = \sqrt{(R_1^2 + (v\Delta t)^2 - 2R_1 v\Delta t\cos\theta)}$$

$$\sim R_1 - (\boldsymbol{v}\cdot\boldsymbol{n})\Delta t$$

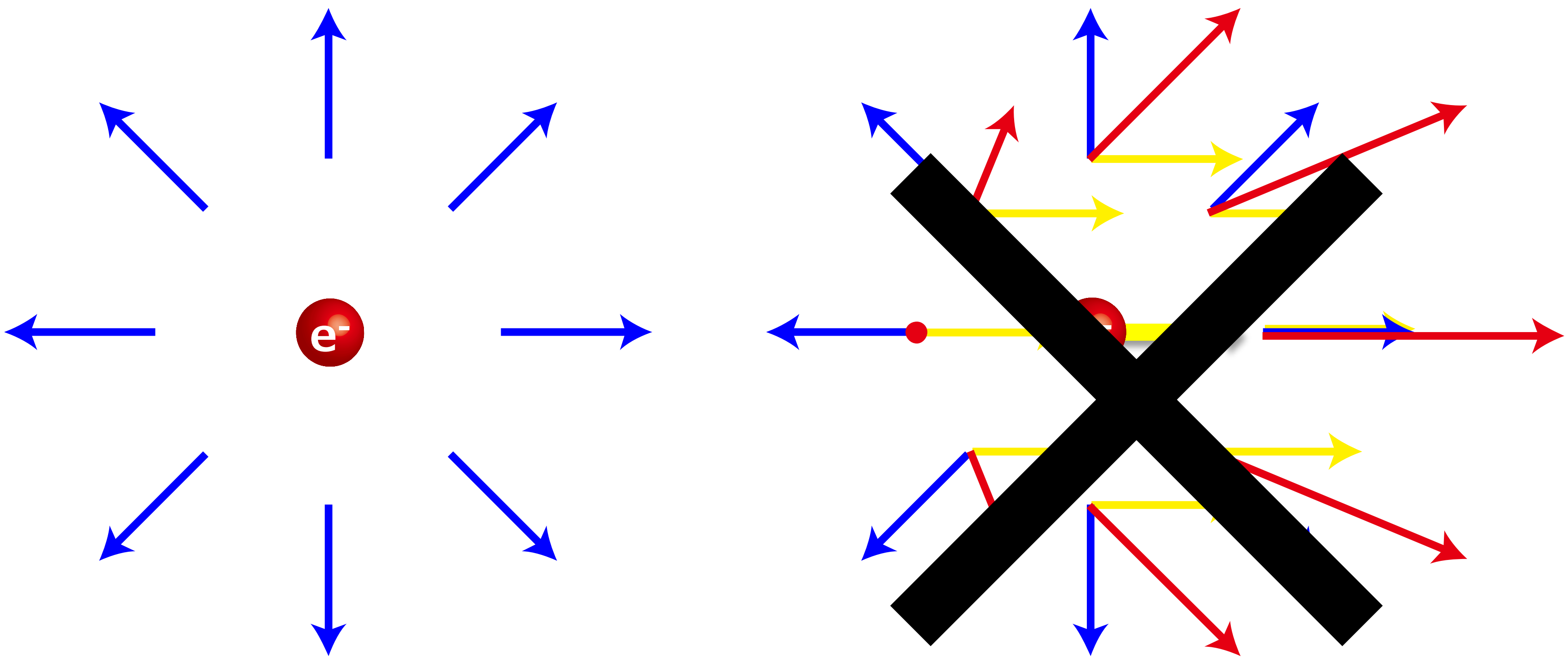
$$\Delta\tau = t_2 - t_1 = \Delta t + R_2/c - R_1/c$$

$$= \Delta t\,(1 - \boldsymbol{\beta}\cdot\boldsymbol{n}) = \Delta t\left(\frac{1}{2\gamma^2} + \theta^2\right)$$

γ>>1,θ~0

Time Squeezing

Photons Emitted by a Moving Electron

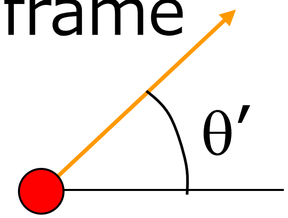


Rest Electron

Moving Electron ($v \sim c$)

Conversion of Emission Angles

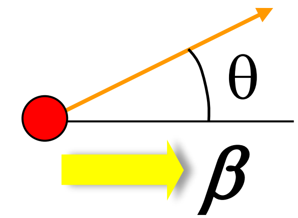
rest frame

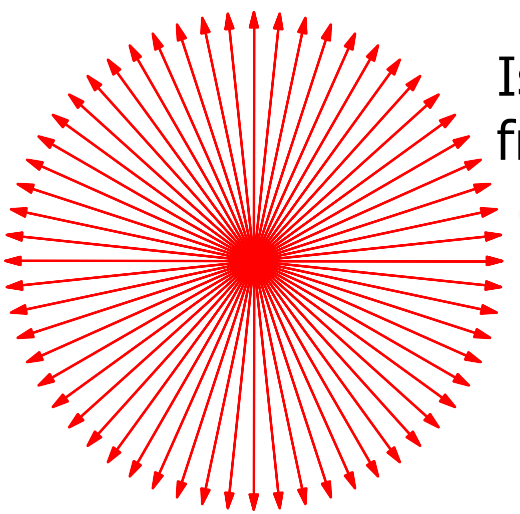


how to convert?

$$\theta = \tan^{-1}\left(\frac{\gamma^{-1}\sin\theta'}{\beta + \cos\theta'}\right)$$

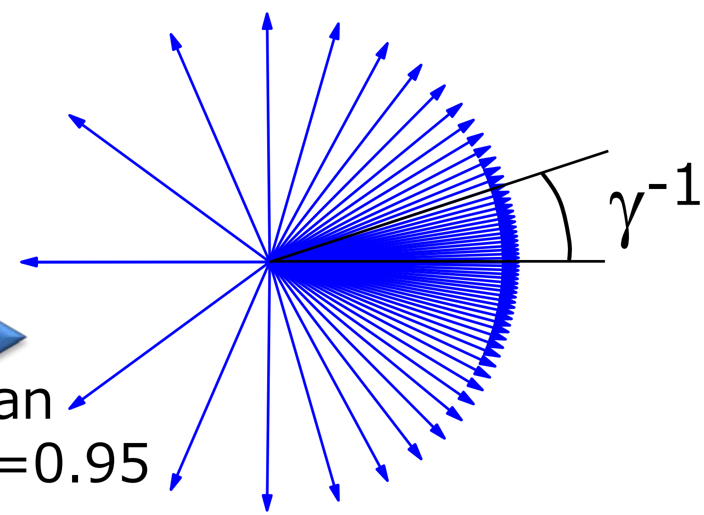
lab. frame





Isotropic emission from a rest electron

Emission from an electron with β=0.95



Light emitted from a moving object (β~1) concentrates within γ^{-1}

SR from a High-Energy Electron

High Energy Electron

$$\beta \sim 1 - \frac{1}{2\gamma^2}$$

Large Time Squeezing

Short Wavelength

$$\theta \leq \gamma^{-1} \ll 1$$

Radiation in the Forward Direction

High Directivity