Light Sources II

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Outline

• Introduction
• Fundamentals of Light and SR
• **Overview of SR Light Source**
  • Characteristics of SR (1)
  • Characteristics of SR (2)
• Practical Knowledge on SR
What is SR Light Source?

Magnets to deflect the electron beam and generate SR.

Bending Magnet (BM)

Insertion Device (ID)
Bending Magnet

- One of the accelerator components in the storage ring.
- Generate **uniform field** to guide the electron beam into a **circular orbit**.
- EMs combined with highly-stable power supplies are adopted in most BMs to satisfy the stringent requirement on field quality and stability.
- Superconducting magnets are used in a few facilities in pursuit of harder x rays.
Insertion Device

- Installed (inserted) into the straight section of the storage ring between two adjacent BMs.
- Generate a periodic magnetic field to let the injected electron beam move along a periodic trajectory.
- Most IDs are composed of PMs, while EMs are used for special use such as helicity switching.
- Two types: wiggler and undulator
In each type, a sinusoidal magnetic field is obtained:

\[ B_y(z) \sim B_0(B_r, g/\lambda_u) \sin \left( \frac{2\pi z}{\lambda_u} \right) \]
Example of ID Magnets

Halbach-type Magnet Array for SPring-8 Standard Undulators

e- Beam

NdFeB Magnet

cooling channel
Example of SR Image

BL41XU@SP-8, first image of SR with a fluorescent screen (<0.1mA)

SR from Downstream BM

SR from Undulator

SR from Upstream BM

\[ g = 50 \text{ mm} \]

\[ g = 20 \text{ mm} \]
Comparison of SR Light Sources

- **Photon Flux Density**
  - (photons/sec/mrad^2/0.1%b.w.)

- **Photon Energy (eV)**

- **Bending Magnet**
- **Undulator (BL09XU)**
- **Wiggler (BL08W)**
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• Characteristics of SR (1)
  – Radiation from Bending Magnets

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Directionality of BM Radiation

- Directional (V-plane)
- Isotropic (H-plane)

BM radiation is similar to a beacon lamp:
- Sheet beam
- 2D directionality
Pulse Structure of BM Radiation

\[ \gamma^{-1} \]
Pulse Structure of BM Radiation

\[ \Delta \tau \]

- Time
- Intensity
What’s the Pulse Duration?

Pulse duration for e-:
\[ \Delta t = \frac{2\rho \gamma^{-1}}{c} \]

Time Squeezing

Pulse duration for observer:
\[ \Delta \tau = \frac{\Delta t}{2\gamma^2} = \frac{\rho}{\gamma^3 c} \]
BM radiation has a white spectrum reaching the frequency of \( \Delta \omega \sim 1/\Delta \tau \sim \gamma^3 c/\rho \)

\[
\omega_c = \frac{3}{2\Delta \tau} = \frac{3\gamma^3 c}{2\rho}
\]

(“critical frequency”) gives a criterion for BM spectrum.

In practical units,

\[
\hbar \omega_c (\text{keV}) = 0.665 E^2 (\text{GeV}^2) B (\text{T})
\]
Example of Spectrum

- **Photon Flux (photons/sec/0.1% B.W.)**
- **Photon Energy (eV)**

- $\Delta \theta_x = 0.1 \text{ mrad}$
- $\Delta \theta_y = 10 \text{ mrad}$
- $\Delta \theta_y = 0.1 \text{ mrad}$

- Larger aperture collects photons with lower energies

- Critical Energy

![Graph showing photon flux and energy relationship](image)
Angular Profile of BM Radiation

- Power profile ~ flux profile at $\omega/\omega_c$
- Larger angular divergence for lower energy

Empirical formula for angular divergence of BM radiation:

$$\sigma_{yr} = 0.6\gamma^{-1} \sqrt{\omega/\omega_c}$$
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Coordinate Systems

\[ \mathbf{r}(t) = (x_e(t), y_e(t), z_e(t)) \]

\[ \mathbf{R} = (X, Y, Z) \]

Assuming \( |\mathbf{r}| \ll Z \) (far-field approximation), the radiation pattern depends only on \( \Theta = (\theta_x, \theta_y) \)
Field Integrals

\[ \frac{dP}{dt} = m\gamma \frac{dv}{dt} = -ev \times B \]

Equation of motion of an electron moving in a magnetic field \( B \)

\[ m\gamma v_x = -e(v_y B_z - v_z B_y) \]
\[ m\gamma v_y = -e(v_z B_x - v_x B_z) \]

\[ m\gamma \frac{dv_{x,y}}{v_z dt} = m\gamma \frac{dv_{x,y}}{dz} = \pm eB_{y,x} \]

\[ \beta_{x,y} = \pm \frac{e}{\gamma mc} \int^{z} B_{y,x}(z')dz' \equiv \pm \frac{e}{\gamma mc} I_{1y,1x}(z) \]

\[ x_e, y_e = \pm \frac{e}{\gamma mc} \int^{z} dz' \int^{z'} B_{y,x}(z'')dz'' \equiv \pm \frac{e}{\gamma mc} I_{2y,2x}(z) \]

\( I_1, I_2 \): 1st and 2nd field integrals of the ID
Trajectory in an Ideal ID

\[
\begin{align*}
B_x(z) &= 0 \\
B_y(z) &= B_0 \sin \left( \frac{2\pi z}{\lambda_u} \right)
\end{align*}
\]

\[
\begin{align*}
\beta_y(z) &= 0 \\
\beta_x(z) &= \frac{K}{\gamma} \cos \left( \frac{2\pi z}{\lambda_u} \right)
\end{align*}
\]

\[
\begin{align*}
y_e(z) &= 0 \\
x_e(z) &= \frac{\lambda_u K}{2\pi \gamma} \sin \left( \frac{2\pi z}{\lambda_u} \right)
\end{align*}
\]

Magnetic Field $\rightarrow$ Velocity $\rightarrow$ Position

\[
K = \frac{eB_0\lambda_u}{2\pi mc} = 93.37B_0(T)\lambda_u(m)
\]

$\checkmark$ K Value

$\checkmark$ Deflection Parameter

\[
E=8\text{GeV}, K=1, \lambda_u=5\text{cm}: \beta_{x_{\text{max}}} = 64\mu\text{rad}, x_{\text{max}} = 0.5\mu\text{m}
\]
Effects due to the ID Magnetic Field

transverse velocity

\[ \beta_x(z) = \frac{K}{\gamma} \cos \left( \frac{2\pi z}{\lambda_u} \right) \]

longitudinal velocity

\[ \beta_z = \sqrt{\beta^2 - \beta_x^2} \]

\[ = 1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2} - \frac{K^2}{4\gamma^2} \cos \left( \frac{4\pi z}{\lambda_u} \right) \]

Original value

Average Value \( \bar{\beta}_z \)

Oscillating Term

ID field induces:

✓ transverse (x) oscillation
✓ longitudinal (z) oscillation
✓ effective deceleration \( (\Delta \beta_z = K^2 / 4\gamma^2) \)
General Form of Time Squeezing

\[
\frac{d\tau}{dt} = 1 - \beta \cdot n
\]

\[
\beta_z = \sqrt{\beta^2 - \beta_x^2 - \beta_y^2}
\]

\[
\sim 1 - (\gamma^{-2} + \beta_x^2 + \beta_y^2)/2
\]

\[
n_z \sim 1 - (\theta_x^2 + \theta_y^2)/2
\]

\[
= \frac{1}{2\gamma^2} + (\theta_x - \beta_x)^2 + (\theta_y - \beta_y)^2
\]

Time squeezing takes place most significantly when the electron is moving in the direction of observation \((\beta = \theta)\).
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  – Undulator Radiation
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Wiggler Radiation

• Wiggler radiation (WR) is regarded as **incoherent sum of SR** emitted at each position of wiggler.
  – Summation as photons in the framework of geometrical optics.

  \[
  F_w \sim 2NF_{BM} \\
  \sigma_{x',y'} \times \sigma_{x,y} \gg \lambda/4\pi \\
  B_w \ll 2NB_{BM}
  \]
• Larger \( N \) results in larger area of photon distribution in the phase space, i.e., larger emittance.

• \( B \) does not linearly depend on \( N \)
Comparison with BM Radiation

**Photon Energy (eV)**

- BM ($\varepsilon_c = 29\text{keV}$), $N=37$
- Wiggler ($\varepsilon_c = 38\text{keV}$), $N=37$

**Flux Density**

- BM
- Wiggler

**Brilliance**

- BM
- Wiggler
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Fundamental Wavelength of UR

\[ \bar{\beta}_z = 1 - \frac{1 + K^2/2}{2\gamma^2} \]

Time Sequeezing

\[ T' = T(1 - \bar{\beta}_z \cos \theta) \]

Period of observed light

Fundamental Wavelength \( \lambda_1 \)

\[
\begin{align*}
\lambda_1 &= \lambda_u \left(1 - \bar{\beta}_z \cos \theta\right) \\
&= \frac{\lambda_u}{2\gamma^2} \left(1 + \gamma^2 \theta^2 + K^2/2\right) \\
\omega_1 &= \frac{2\pi c}{\lambda_1}
\end{align*}
\]

H. Kitamura et al.,
UR with Infinite Periods

• If the undulator length is infinite, the pulse duration is infinitely long, and thus the radiation is completely monochromatic with line spectrum.

\[ f(\theta, \omega) = \delta(\omega - \omega_1) = \delta \left( \omega - \frac{4\pi c \gamma^2 / \lambda_u}{1 + K^2/2 + \gamma^2 \theta^2} \right) \]

• In practice, the undulator length is finite, so the line spectrum is broadened.
Effects due to Finite Periods

\[ E(t) = \begin{cases} 
E_0 \sin \omega_1 t & ; -T/2 \leq t \leq T/2 \\
0 & ; t < -T/2, \ T/2 < t 
\end{cases} \]

\[ \omega_1 = \frac{2\pi c}{\lambda_1} \]

Fourier Transform

\[ f(\theta, \omega) \propto |\hat{E}(\omega)|^2 \propto \text{sinc}^2 \left[ \pi N \frac{\omega - \omega_1(\theta)}{\omega_1(\theta)} \right] \]

Square of “sinc” function dominates UR
Brief Note on UR Formulae

• In the previous derivations of UR spectral function, no knowledge on electrodynamics is required.

• In practice, $E_0$ is a complicated function of $\theta$ and $K$, and needs to be calculated by Fourier transforming the electric field derived from the Lienard-Wiechercrd potential.

• However, the simple derivation gives us a clear understanding on UR properties.
$f(\theta, \omega) = F_0 \text{sinc}^2 \left[ \pi N \frac{\omega - \omega_1(\theta)}{\omega_1(\theta)} \right]$
Angular Profile of UR

\[
f(\theta, \omega) = F_0 \text{sinc}^2 \left[ \pi N \frac{\omega - \omega_1(\theta)}{\omega_1(\theta)} \right]
\]

\[
\omega = 0.9 \omega_1(0)
\]

\[
\omega = \omega_1(0)
\]

\[
\omega = 1.1 \omega_1(0)
\]
Angular Divergence of UR

**UR is not a Gaussian Beam**

Angular Profile at $\omega = \omega_1(0)$

$$f(\theta, \omega) = F_0 \text{sinc}^2 \left[ \frac{\pi N(\gamma \theta)^2}{1 + K^2/2} \right]$$

**Gaussian Profile with $\sigma_{r'}$**

$$f_G(\theta) = F_0 \exp \left( -\frac{\theta}{2\sigma_{r'}^2} \right)$$

$$\int f(\theta, \omega) d\theta = \int f_G(\theta) d\theta$$

$$\sigma_{r'} = \sqrt{\frac{1 + K^2/2}{4N\gamma^2}} = \sqrt{\frac{\lambda_1}{2L}}$$

“Natural” Angular Divergence of UR $(L = N\lambda_u)$
Source Size of UR

Unlike WR, UR is spatially coherent, i.e., diffraction limited.

This means that \( \sigma_r \sigma_r' = \frac{\lambda}{4\pi} \)

“Natural” Angular Divergence of UR
\( L = N\lambda_u \)

“Natural” Source size of UR

\[
\sigma_r = \frac{\lambda_1}{4\pi \sigma_r'} = \frac{\sqrt{2L\lambda_1}}{4\pi}
\]

 Longer device results in smaller angular divergence & larger source size, but the emittance does not change.
Photons with at $n \omega_1$ are observed as well as at $\omega_1$, where $n$ is an integer.
$K \ll 1$ Case

$K\gamma^{-1}$

Electric Field

Time
$K \sim 1$ Case

$K \gamma^{-1}$

Electric Field

Time
K >> 1 Case

\[ K\gamma^{-1} \]
Mechanisms of Higher Harmonics

\[ K \ll 1 \]
Electron Orbit
Radiation E-field
Spectrum

\[ K \approx 1 \]

\[ K \gg 1 \]
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Effective Properties of SR

- Properties of SR emitted from an e\(^{-}\) beam are different from those from a single e\(^{-}\).
- They are referred to as “effective” properties of SR, as opposed to “natural” properties.
Example in SPring-8: E-t Phase Space

- **Electron Beam**: 20 psec, 0.1% Energy, 0.05 psec
- **Undulator Radiation**: 1% Energy, 100% Intensity
- **BM Radiation**: 0.7 fsec, 100% Intensity
Example in SPring-8: (x,y) Space

Electron Beam

Undulator Radiation

BM Radiation
Example in SPring-8: $(x',y')$ Space
Convolution Between e- and Photon

Electron distribution

Photon distribution from a single e-

Photon distribution from e- beam

Convolution

Broadening of temporal (t), spectral (ω), spatial (x,y) and angular profiles (x’,y’).
Spectral Profile (UR)

Emittance (m.rad)
- $10^{-10}$
- $10^{-9}$
- $10^{-8}$
- $10^{-7}$

Photon Flux Density (arb.unit)

E-spread (%)
- 0.001
- 0.01
- 0.1
- 1

Relative Photon Energy ($\omega_1$)
Angular & Spatial Profile (UR)

- Gauss approximation
- Convolution theorem

Angular profile of UR from a single electron

Angular profile of UR from the e-beam

Convolution

Angular profile of the e-beam

- Effective Angular Div.:
  \[ \Sigma_{x',y'} = \sqrt{\sigma_{r'}^2 + \sigma_{x',y'}^2} \]

- Effective Source Size:
  \[ \Sigma_{x,y} = \sqrt{\sigma_r^2 + \sigma_{x,y}^2} \]
Heat Load on Optical Elements

• SR is usually processed by several optical elements before irradiation to the sample, such as the focusing mirror, monochromator.

• These elements can be easily damaged by the heat load of SR.

• In the case of UR, the heat load can be reduced by taking advantage of the difference in the angular profile of the photon flux and radiation power.
The power profile is much broader than the flux. Extraction of SR with an appropriate slit is thus effective.
What’s the Optimum Aperture Size?

![Graph showing the relationship between slit size and photon flux and power](image)
Wiggler? Undulator? (1)

• Wigglers are identical to undulator from the point of view of magnetic circuit.
• It is generally said that the K value distinguishes between the two, however, this is not exactly correct.
• What we should take care is the photon energy region of interest.
Wiggler? Undulator? (2)

Photon Flux (photons/sec/0.1%B.W.)

Photon Energy (eV)

undulator  wiggler

$\lambda_u = 12\text{cm}$

$K = 10$

$\Delta x = \Delta y = 3\text{mm} @ 30\text{m}$
Undulator Radiation Gallery

• For quantitative evaluation of SR, a computer code “SPECTRA” is available.
• SPECTRA also offers a function to “visualize” the computation results for further understanding of SR.
  – brilliance curve & spectrum
  – on- and off-peak angular profiles of flux
  – on- and off-axis spectra
  – effects of opening the slit aperture
  – undulator-to-wiggler transition
Brilliance Curve & Spectrum

Spectrum –, Peak Brilliance $1^{\text{st}}$ – $3^{\text{rd}}$ – $5^{\text{th}}$ –

Energy

Brilliance

$10^{14}$

$10^{15}$

$10^{16}$

$10^{17}$

$10^{18}$

$10^{19}$

$10^{20}$

$10^3$ – $10^4$ – $10^5$
Opening the Slit Aperture

Flux vs. Energy Graph

Aperture/BeamSize = 0.1
Off-Axis Spectrum

Moving the slit along - x, - y
Flux Angular Profile

On-Axis Spectrum

Angular Profile (Finite Emittance)

Angular Profile (Zero Emittance)
Angular Divergence & Source Size

\[ \frac{\rho}{2} \left( \frac{\Delta \theta}{2} \right)^2 \sim 2\sigma_x \]

\[ \Delta \theta \sim 2\sigma_x \]

\[ \rho \Delta \theta \]

\[ \sigma_{yf} = \frac{\rho \Delta \theta}{2} \sigma_y \]

\[ \sigma_{yd} = \frac{\lambda}{4\pi \sigma_{yf}} \]

\[ \sigma_y \sim \sqrt{\sigma_{yd}^2 + \sigma_{yf}^2} \]