Light Sources II

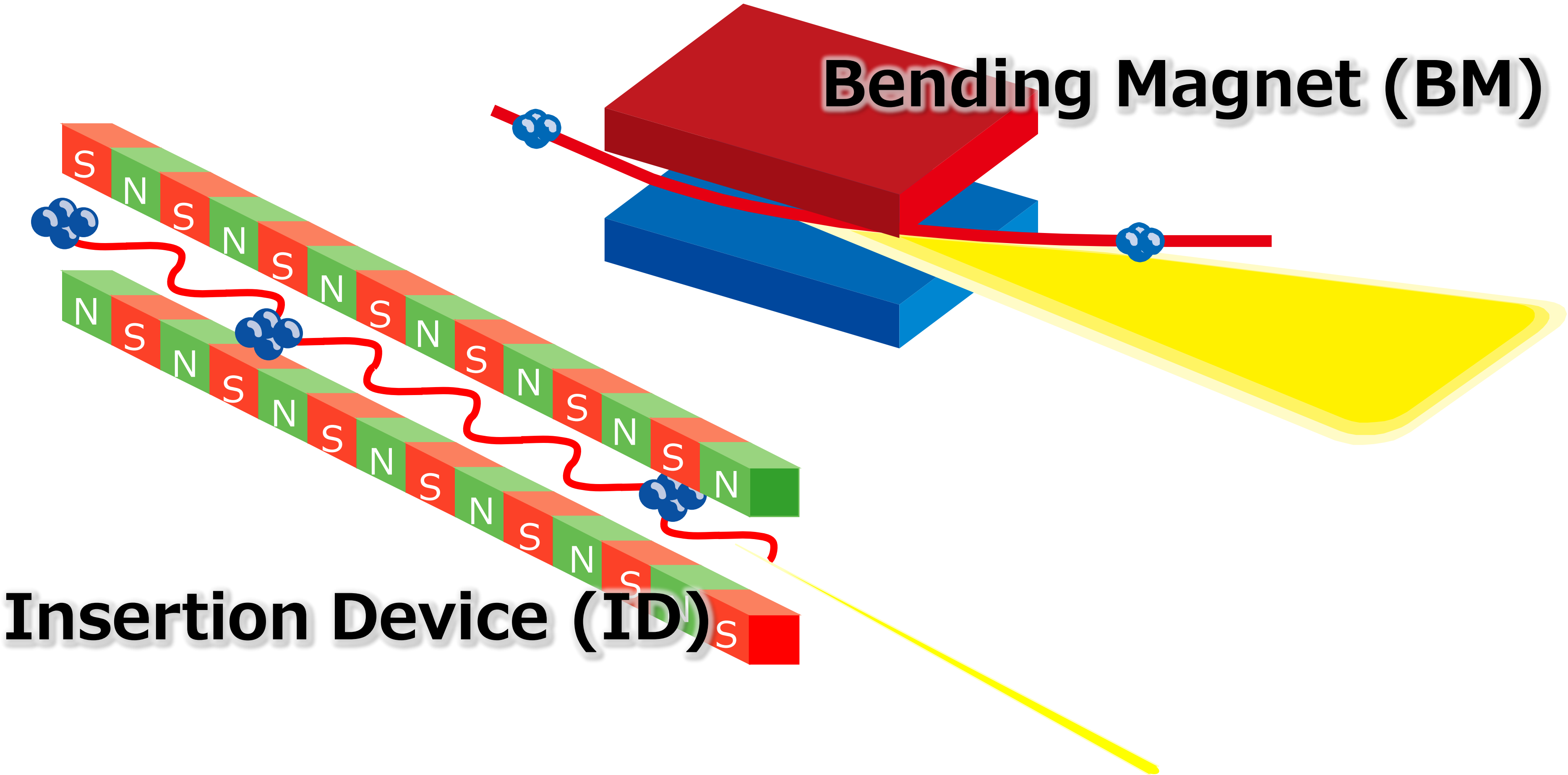
Takashi TANAKA
RIKEN SPring-8 Center

Outline

- Introduction
- Fundamentals of Light and SR
- **Overview of SR Light Source**
- Characteristics of SR (1)
- Characteristics of SR (2)
- Practical Knowledge on SR

What is SR Light Source?

Magnets to deflect the electron beam and generate SR.



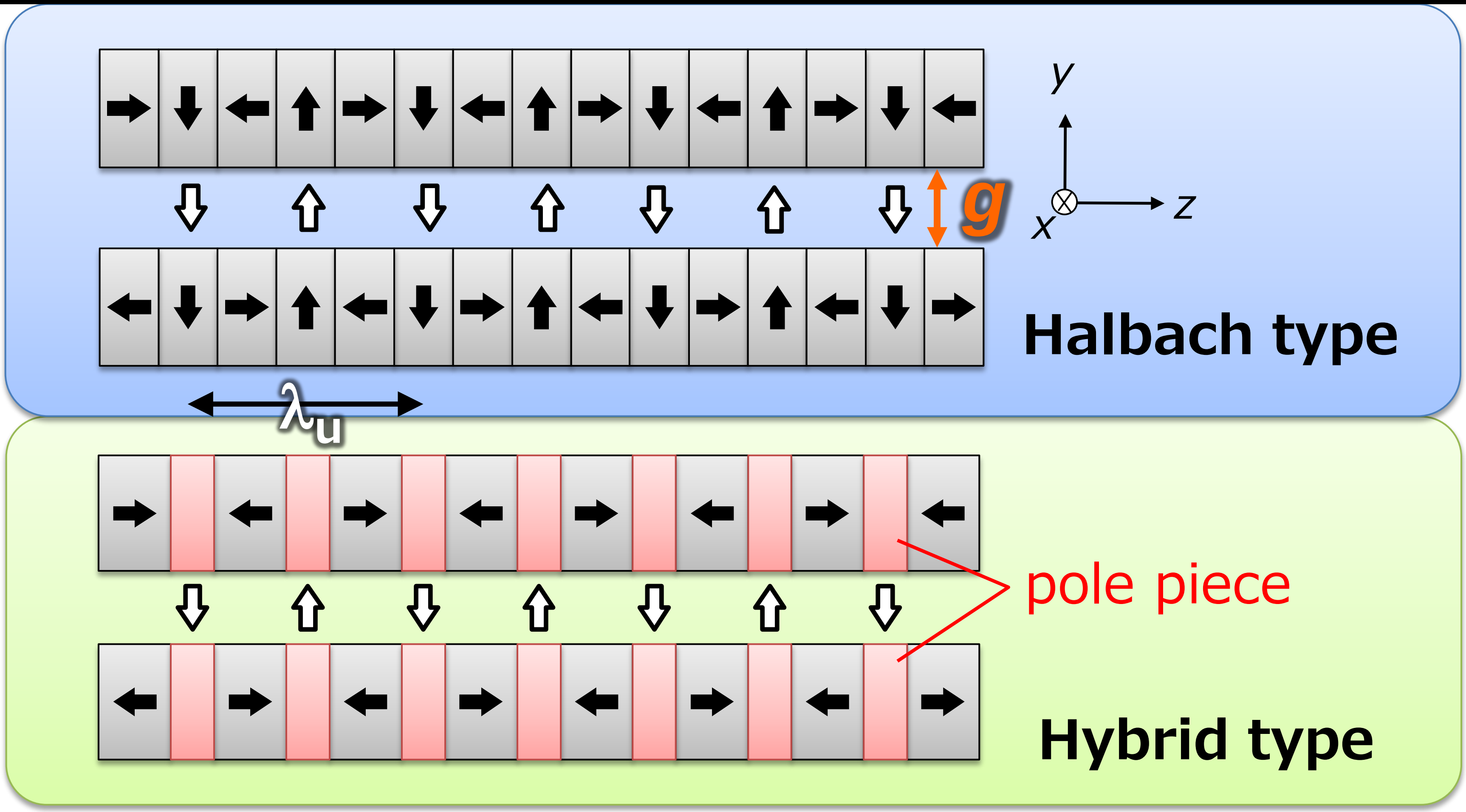
Bending Magnet

- One of the accelerator components in the storage ring.
- Generate **uniform field** to guide the electron beam into a **circular orbit**.
- EMs combined with highly-stable power supplies are adopted in most BMs to satisfy the stringent requirement on field quality and stability.
- Superconducting magnets are used in a few facilities in pursuit of harder x rays.

Insertion Device

- Installed (inserted) into the straight section of the storage ring between two adjacent BMs.
- Generate a **periodic magnetic field** to let the injected electron beam move along a **periodic trajectory**.
- Most IDs are composed of PMs, while EMs are used for special use such as helicity switching.
- Two types: **wiggler** and **undulator**

Magnetic Circuit of IDs

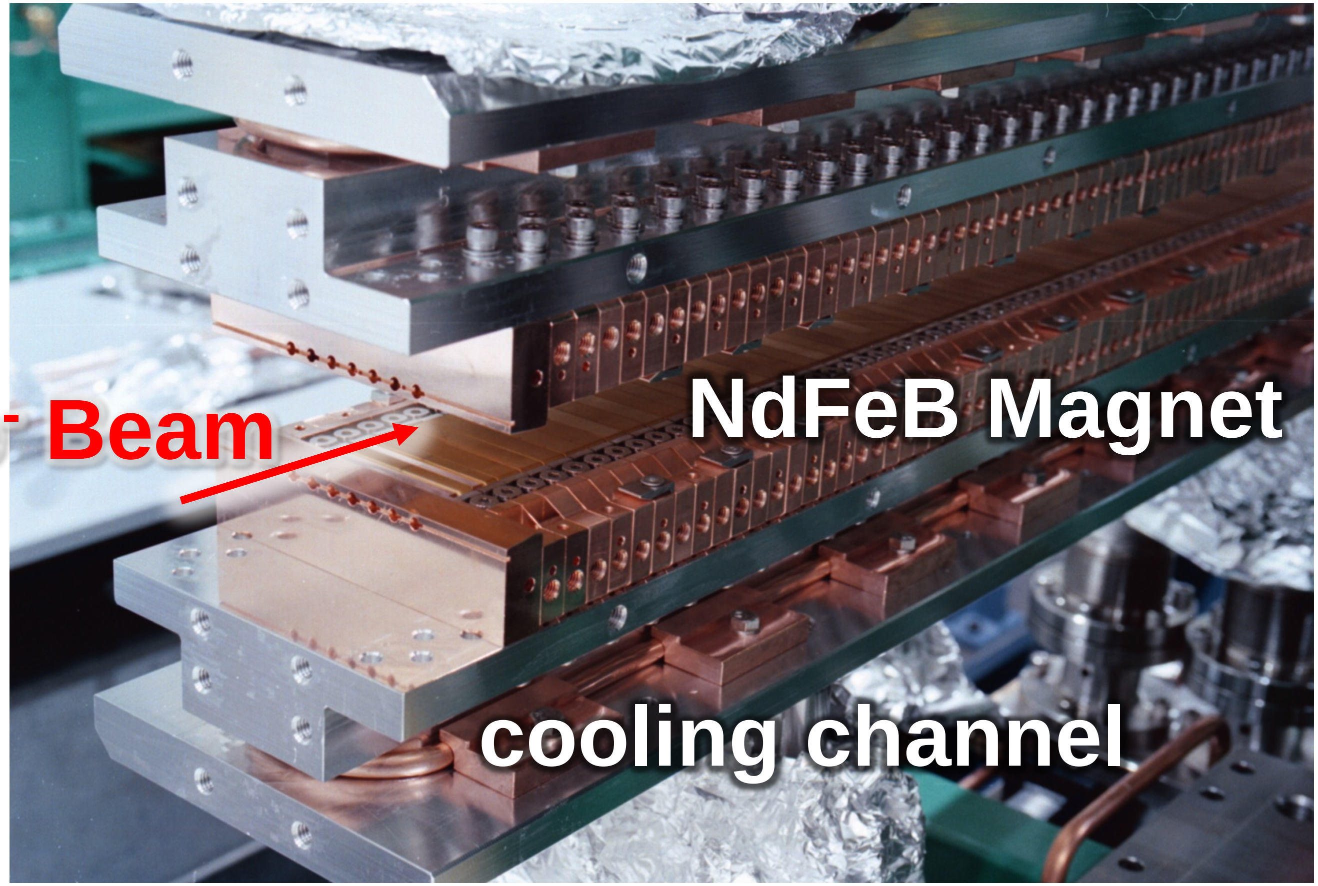


In each type, a sinusoidal magnetic field is obtained:

$$B_y(z) \sim B_0(B_r, g/\lambda_u)\sin\left(\frac{2\pi z}{\lambda_u}\right)$$

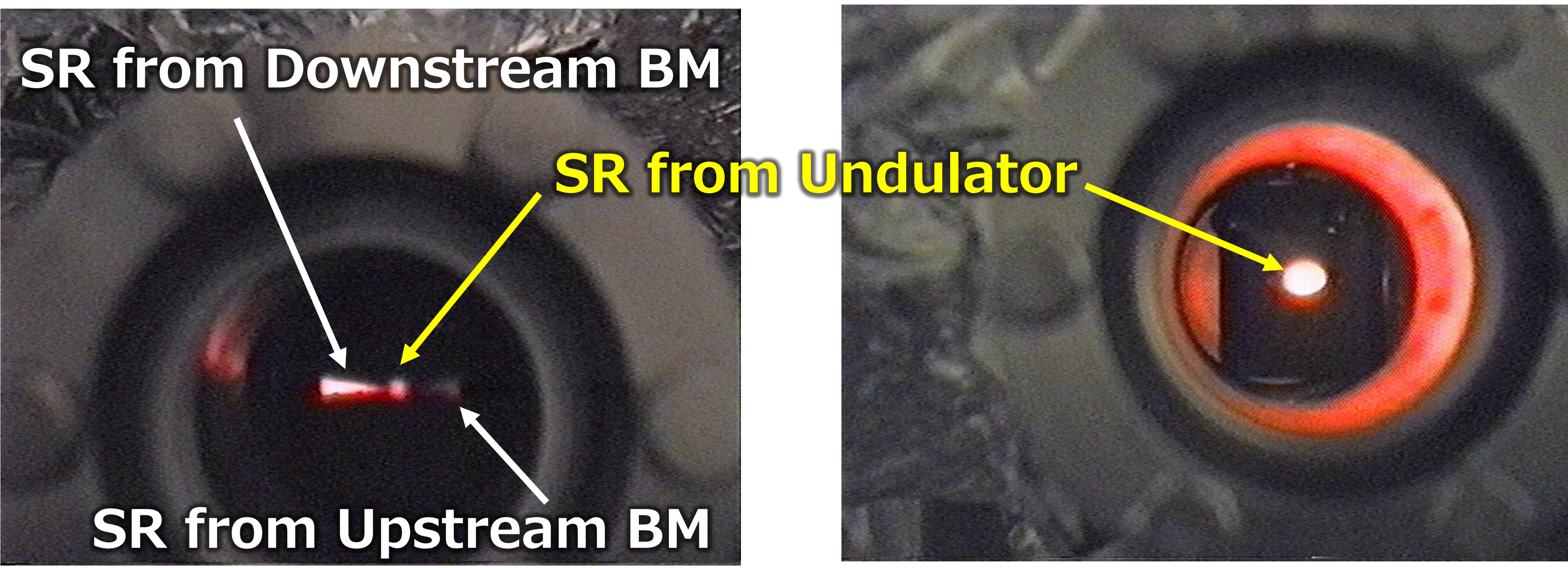
Example of ID Magnets

Halbach-type Magnet Array for
SPring-8 Standard Undulators



Example of SR Image

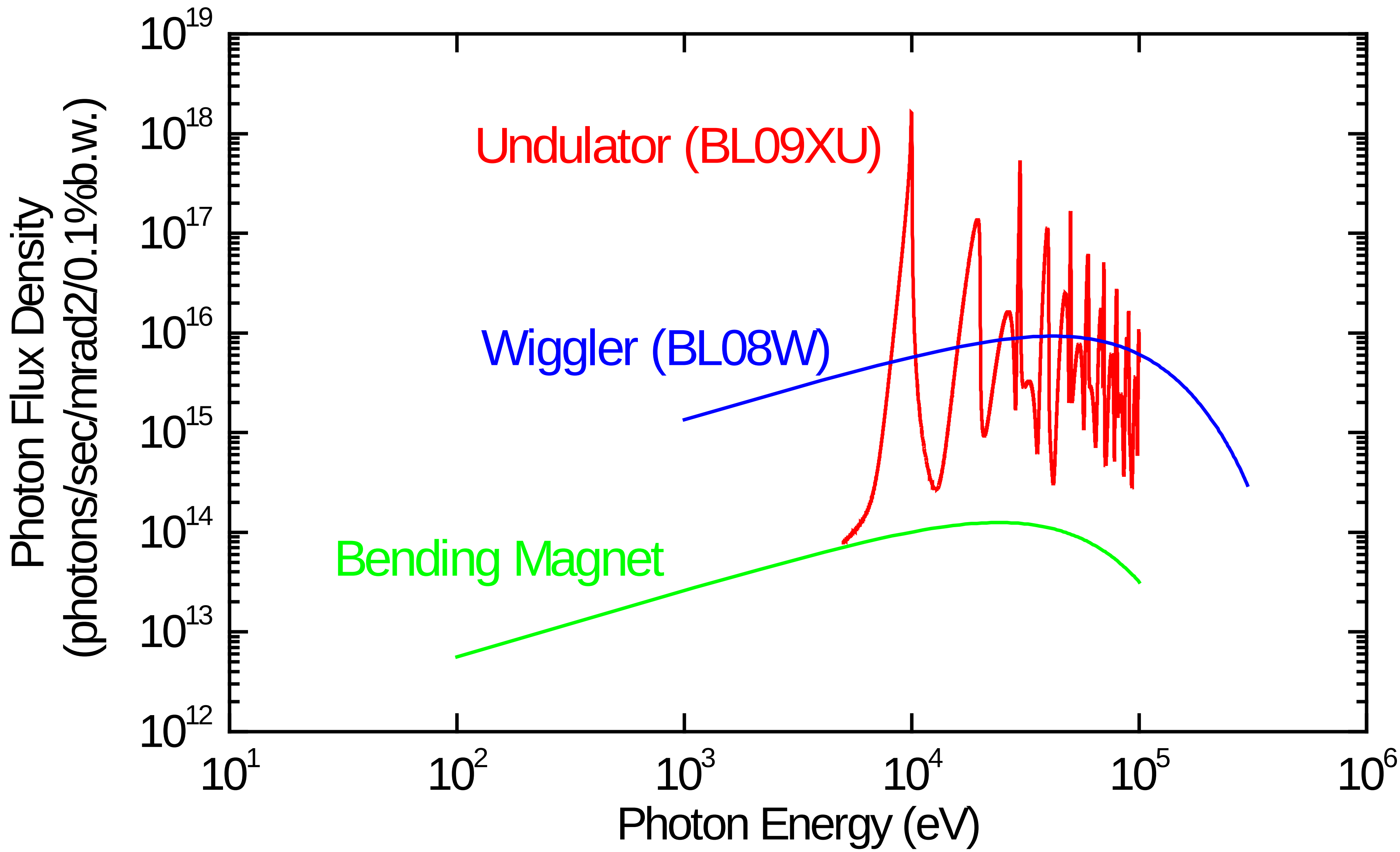
BL41XU@SP-8, first image of SR with a fluorescent screen (<0.1mA)



***g* = 50 mm**

***g* = 20 mm**

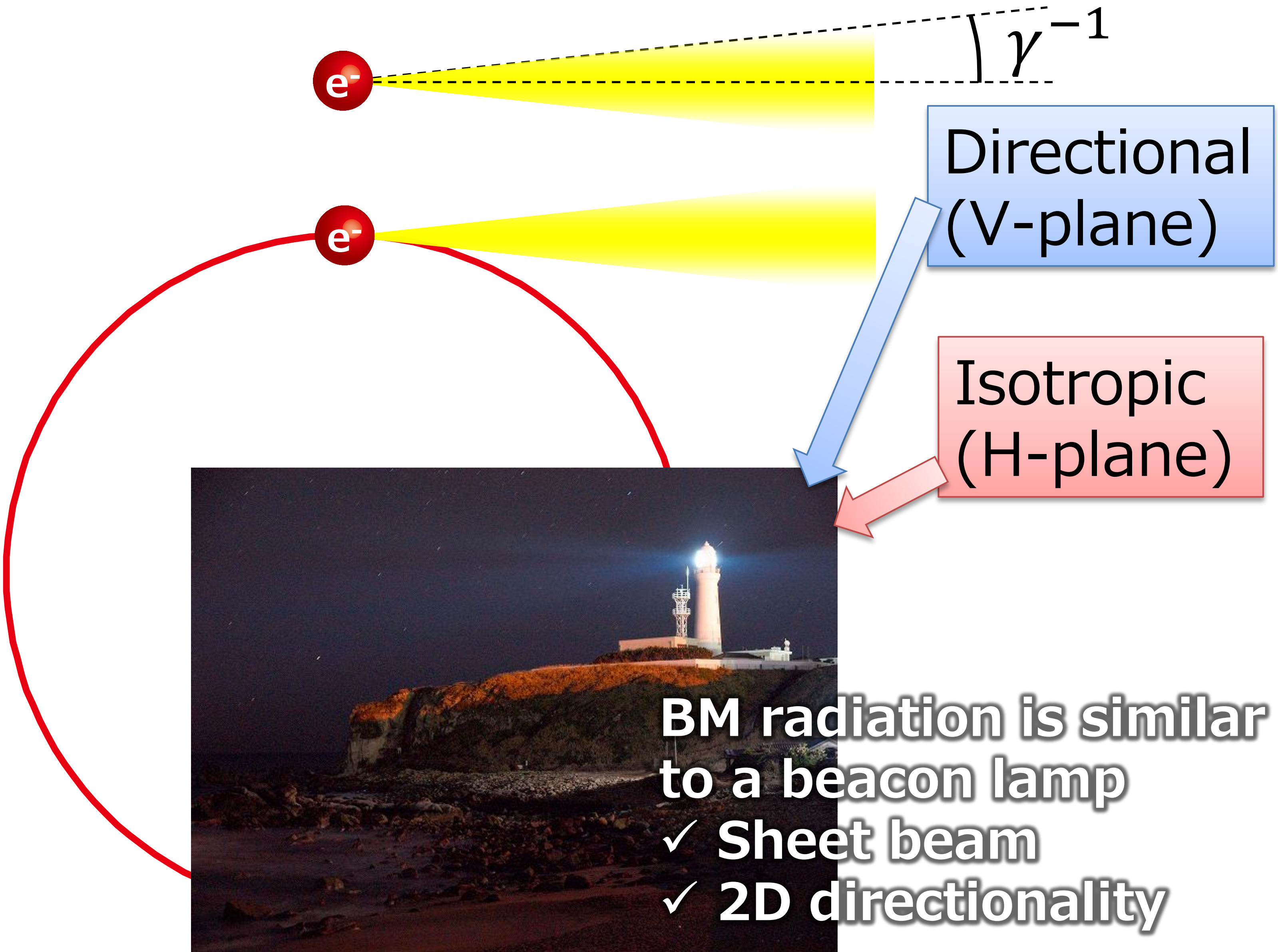
Comparison of SR Light Sources



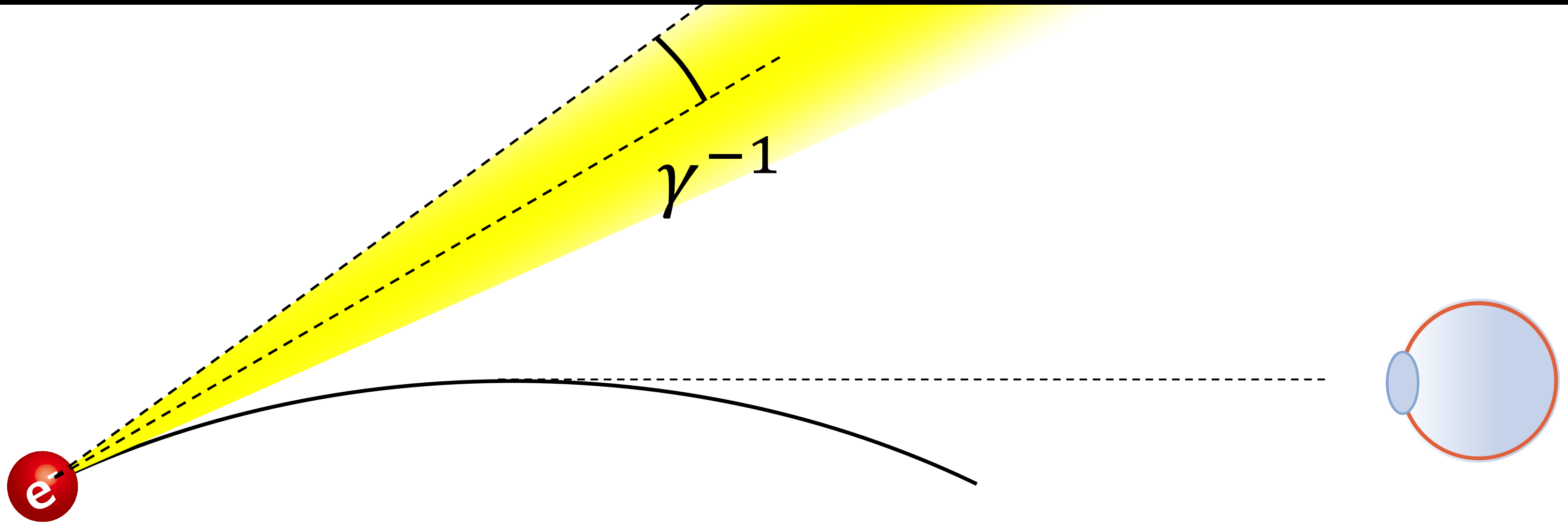
Outline

- Introduction
- Fundamentals of Light and SR
- Overview of SR Light Source
- **Characteristics of SR (1)**
  - **Radiation from Bending Magnets**
- Characteristics of SR (2)
- Practical Knowledge on SR

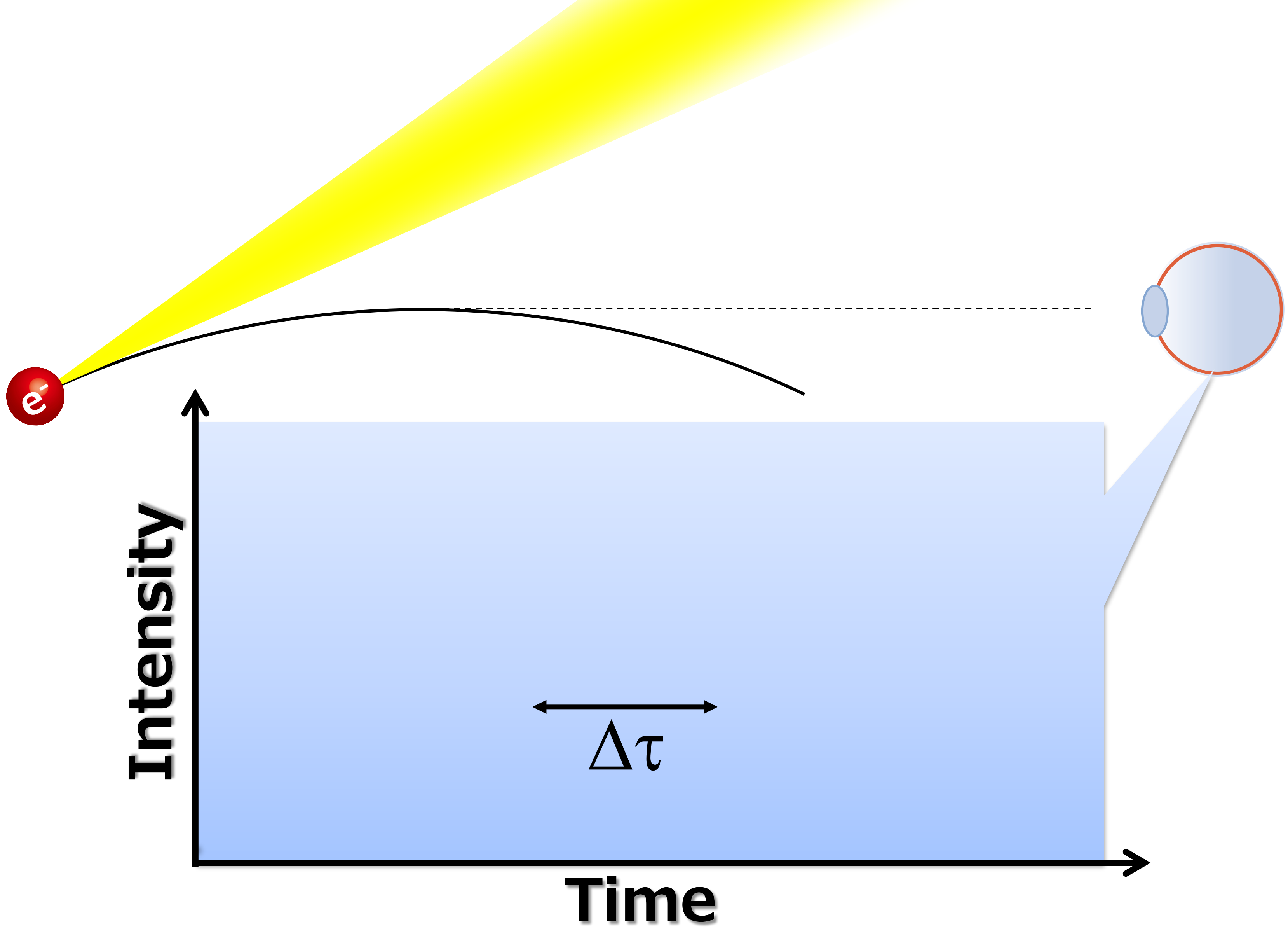
Directionality of BM Radiation



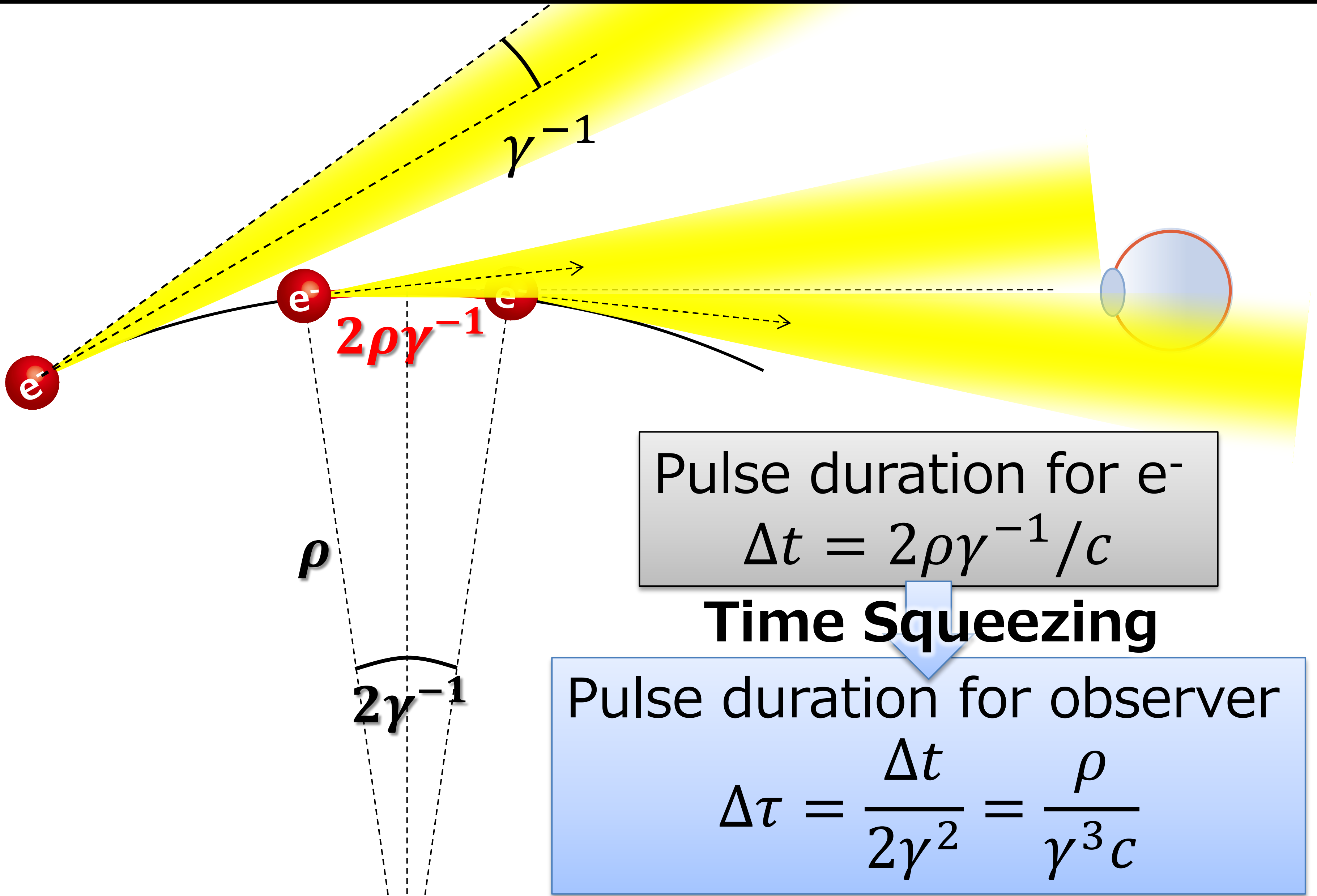
Pulse Structure of BM Radiation



Pulse Structure of BM Radiation



What's the Pulse Duration?



Pulse duration for e⁻

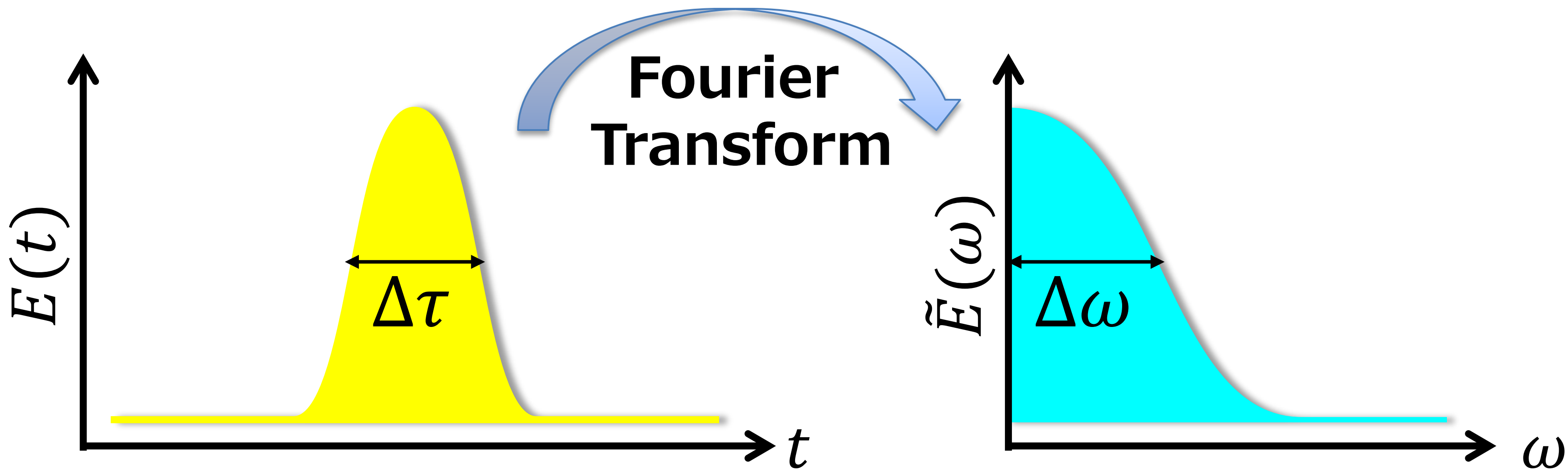
$$\Delta t = 2\rho\gamma^{-1}/c$$

Time Squeezing

Pulse duration for observer

$$\Delta\tau = \frac{\Delta t}{2\gamma^2} = \frac{\rho}{\gamma^3 c}$$

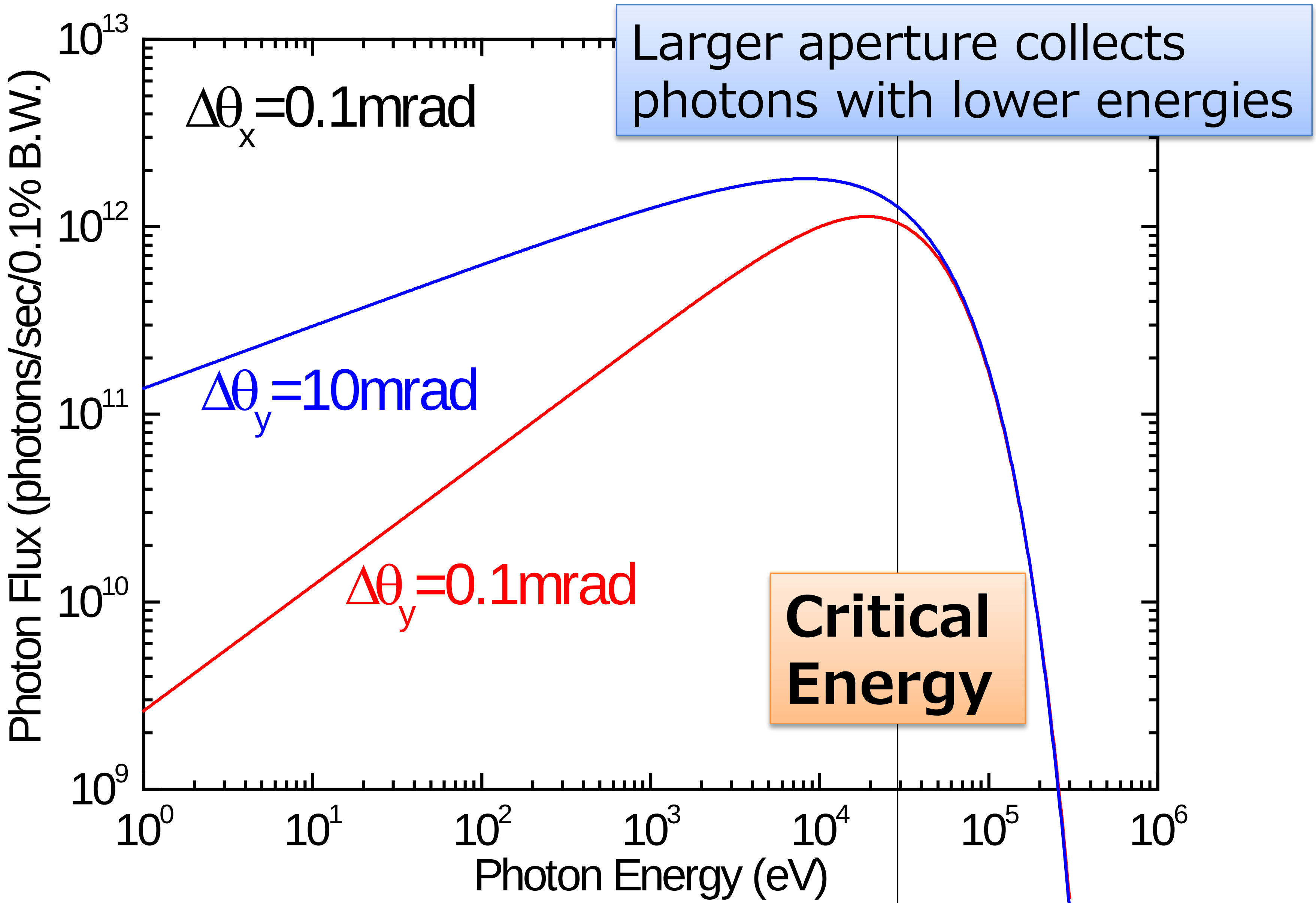
Spectrum of BM Radiation



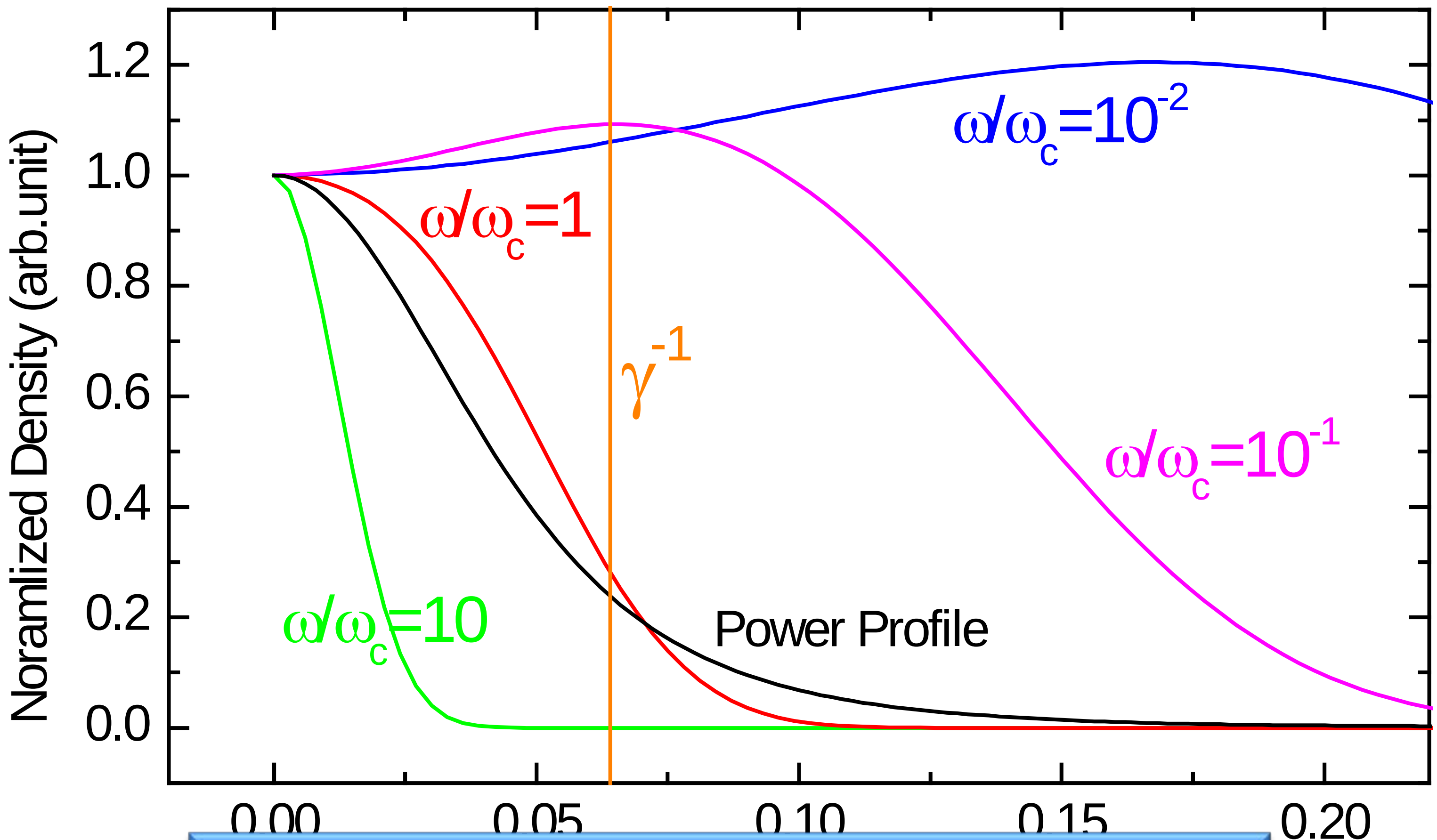
- ✓ BM radiation has a white spectrum reaching the frequency of $\Delta\omega \sim 1/\Delta\tau \sim \gamma^3 c/\rho$
- ✓ $\omega_c = \frac{3}{2\Delta\tau} = \frac{3\gamma^3 c}{2\rho}$ ("critical frequency") gives a criterion for BM spectrum.
- ✓ In practical units,

$$\hbar\omega_c(\mathrm{keV}) = 0.665E^2\ (\mathrm{GeV}^2)B(\mathrm{T})$$

Example of Spectrum



Angular Profile of BM Radiation



Empirical formula for angular divergence of BM radiation

$$\sigma_{y'} = 0.6\gamma^{-1}\sqrt{\omega/\omega_c}$$

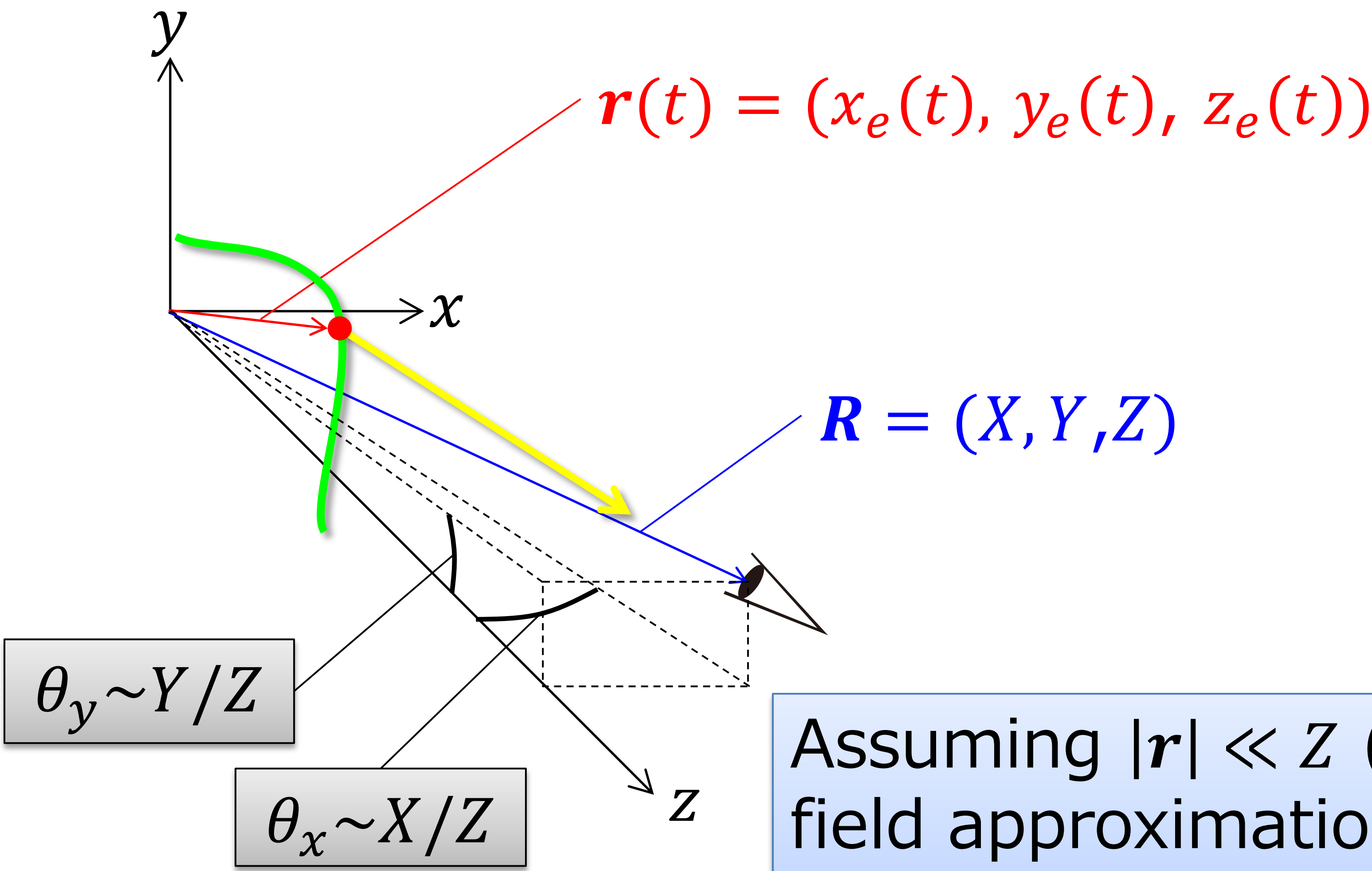
✓ pow

✓ large ... energy

Outline

- Introduction
- Fundamentals of Light and SR
- Overview of SR Light Source
- Characteristics of SR (1)
- **Characteristics of SR (2)**
  - **Electron Trajectory in IDs**
  - Wiggler Radiation
  - Undulator Radiation
- Practical Knowledge on SR

Coordinate Systems



Assuming $|\boldsymbol{r}| \ll Z$ (far-field approximation), the radiation pattern depends only on $\boldsymbol{\Theta} = (\theta_x, \theta_y)$

Field Integrals

$$\frac{d\boldsymbol{P}}{dt} = m\gamma\frac{d\boldsymbol{v}}{dt} = -e\boldsymbol{v}\times\boldsymbol{B}$$

$$\begin{cases} m\gamma\dot{v}_x = -e(v_yB_z - v_zB_y) \\ m\gamma\dot{v}_y = -e(v_zB_x - v_xB_z) \end{cases}$$

Equation of motion of an electron moving in a magnetic field ***B***

$$m\gamma\frac{dv_{x,y}}{v_z dt} = m\gamma\frac{dv_{x,y}}{dz} = \pm eB_{y,x}$$

$$\beta_{x,y} = \pm\frac{e}{\gamma mc}\int^z B_{y,x}(z')dz' \equiv \pm\frac{e}{\gamma mc}I_{1y,1x}(z)$$

$$x_e, y_e = \pm\frac{e}{\gamma mc}\int^z dz'\int^{z'} B_{y,x}(z'')dz'' \equiv \pm\frac{e}{\gamma mc}I_{2y,2x}(z)$$

I_1, I_2: 1st and 2nd field integrals of the ID

Trajectory in an Ideal ID

$$\begin{cases} B_x(z) = 0 \\ B_y(z) = B_0 \sin\left(\dfrac{2\pi z}{\lambda_u}\right) \end{cases}$$

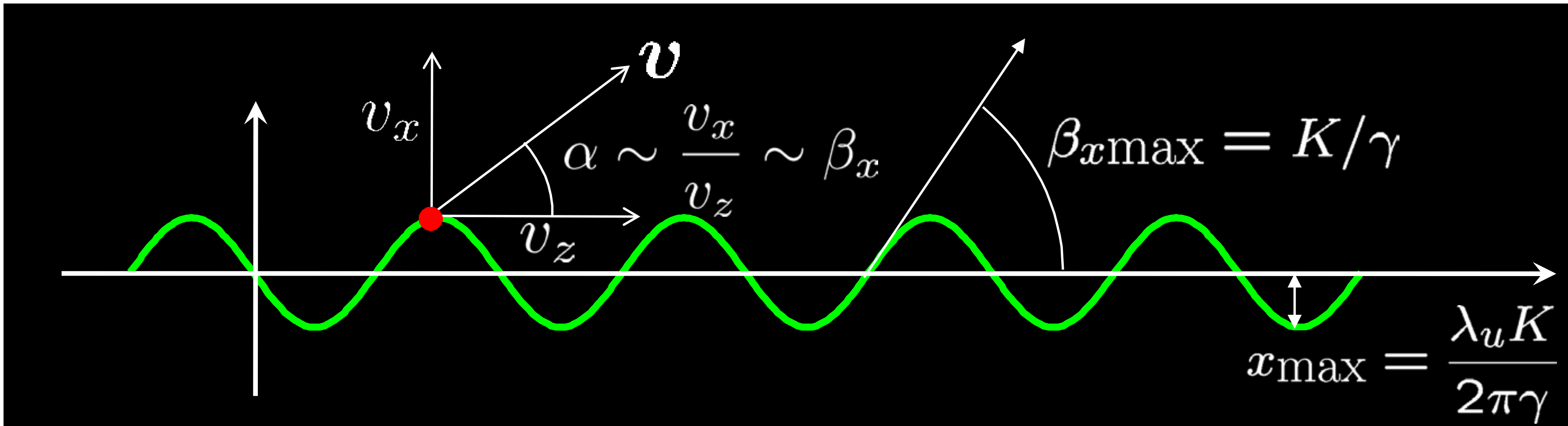
$$\begin{cases} \beta_y(z) = 0 \\ \beta_x(z) = \dfrac{K}{\gamma} \cos\left(\dfrac{2\pi z}{\lambda_u}\right) \end{cases}$$

$$\begin{cases} y_e(z) = 0 \\ x_e(z) = \dfrac{\lambda_u K}{2\pi\gamma} \sin\left(\dfrac{2\pi z}{\lambda_u}\right) \end{cases}$$

Magnetic Field ⇒ **Velocity** ⇒ **Position**

$$K = \frac{eB_0\lambda_u}{2\pi mc} = 93.37 B_0(\mathrm{T})\lambda_u(\mathrm{m})$$

- ✓ **K Value**
- ✓ **Deflection Parameter**



E=8GeV,K=1,λ_u=5cm : β_{xmax}=64μrad, x_{max}=0.5μm

Effects due to the ID Magnetic Field

transverse velocity $$\beta_x(z) = \frac{K}{\gamma}\cos\left(\frac{2\pi z}{\lambda_u}\right)$$

longitudinal velocity
$$\beta_z = \sqrt{\beta^2 - \beta_x^2}$$
$$= 1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2} - \frac{K^2}{4\gamma^2}\cos\left(\frac{4\pi z}{\lambda_u}\right)$$

Original value: $1 - \frac{1}{2\gamma^2}$

Average Value $\overline{\beta_z}$: $1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2}$

Oscillating Term: $- \frac{K^2}{4\gamma^2}\cos\left(\frac{4\pi z}{\lambda_u}\right)$

ID field induces:
- ✓ transverse (x) oscillation
- ✓ longitudinal (z) oscillation
- ✓ effective deceleration ($\Delta\beta_z = K^2/4\gamma^2$)

General Form of Time Squeezing

$$\frac{d\tau}{dt} = 1 - \boldsymbol{\beta} \cdot \boldsymbol{n}$$

$$\begin{aligned} \beta_z &= \sqrt{\beta^2 - \beta_x^2 - \beta_y^2} \\ &\sim 1 - (\gamma^{-2} + \beta_x^2 + \beta_y^2)/2 \\ n_z &\sim 1 - (\theta_x^2 + \theta_y^2)/2 \end{aligned}$$

$$= \frac{1}{2\gamma^2} + (\theta_x - \beta_x)^2 + (\theta_y - \beta_y)^2$$

Time squeezing takes place most significantly when the electron is moving in the direction of observation ($\boldsymbol{\beta} = \boldsymbol{\theta}$).

Outline

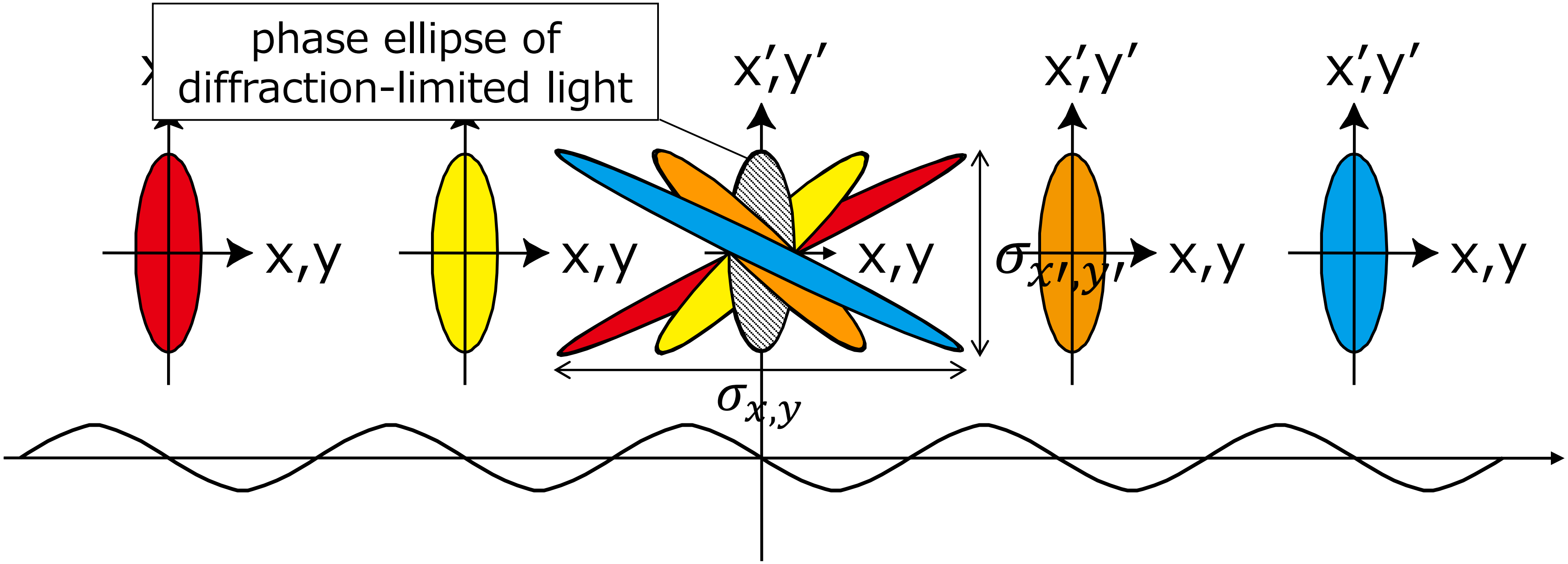
- Introduction
- Fundamentals of Light and SR
- Overview of SR Light Source
- Characteristics of SR (1)
- **Characteristics of SR (2)**
  - Electron Trajectory in IDs
  - **Wiggler Radiation**
  - Undulator Radiation
- Practical Knowledge on SR

Wiggler Radiation

- Wiggler radiation (WR) is regarded as **incoherent sum of SR** emitted at each position of wiggler.
  - Summation as photons in the framework of geometrical optics.

Flux	:	$F_w \sim 2NF_{BM}$
Emittance	:	$\sigma_{x\prime,y\prime} \times \sigma_{x,y} \gg \lambda/4\pi$
Brilliance	:	$B_w \ll 2NB_{BM}$

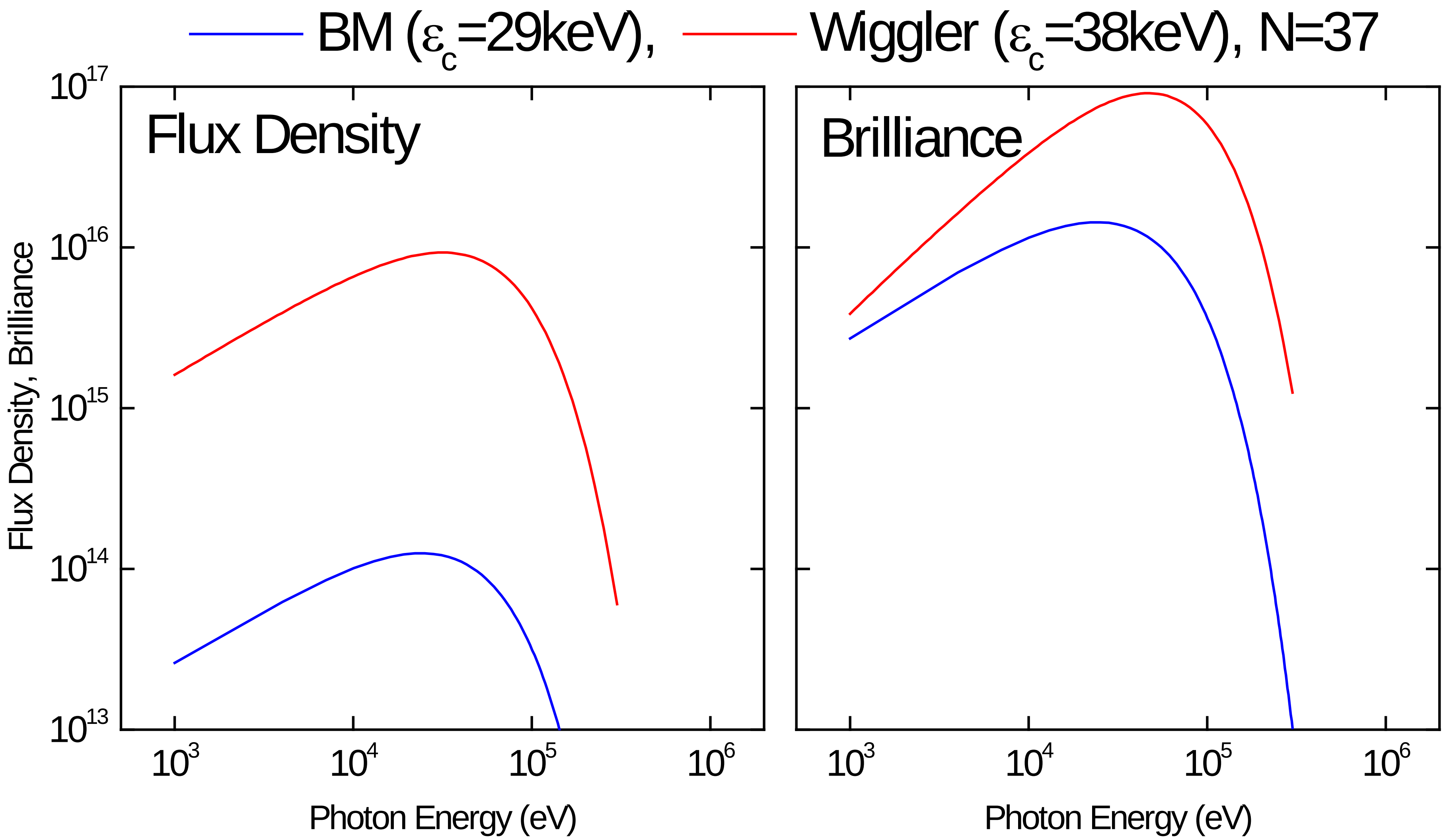
Photon Distribution in Phase Space



Beam Waist Position

- Larger *N* results in larger area of photon distribution in the phase space, i.e., larger emittance.
- *B* does not linearly depend on *N*

Comparison with BM Radiation

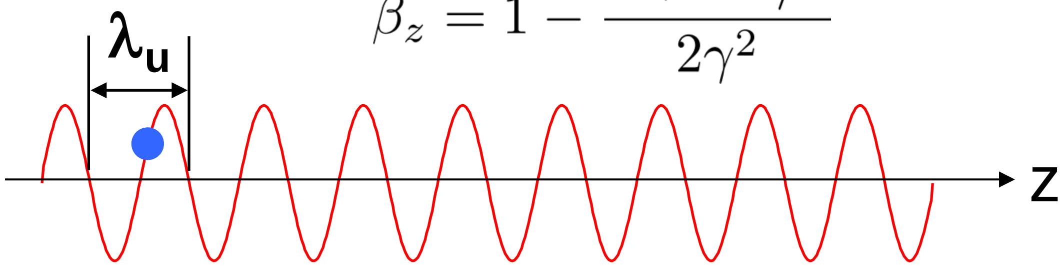


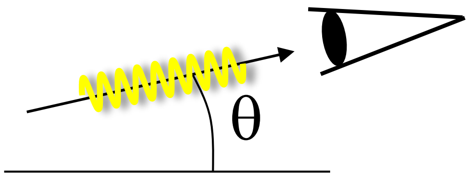
Outline

- Introduction
- Fundamentals of Light and SR
- Overview of SR Light Source
- Characteristics of SR (1)
- **Characteristics of SR (2)**
  - Electron Trajectory in IDs
  - Wiggler Radiation
  - **Undulator Radiation**
- Practical Knowledge on SR

Fundamental Wavelength of UR

$$\bar{\beta}_z = 1 - \frac{1+K^2/2}{2\gamma^2}$$





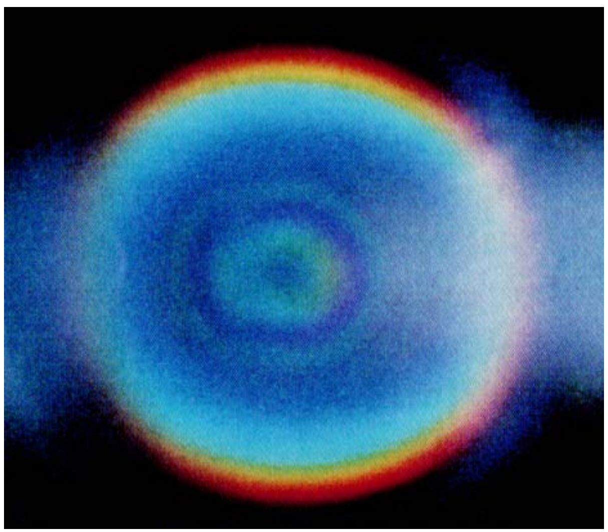
$$T = \lambda_u/v_z = \lambda_u/c$$

period of electron motion
= period of emitted light

Time Sequeezing

$$T' = T(1-\bar{\beta}_z\cos\theta)$$

period of observed light



H. Kitamura et al.,
J. Appl. Phys. 21 (1982) 1728

Fundamental Wavelength λ_1

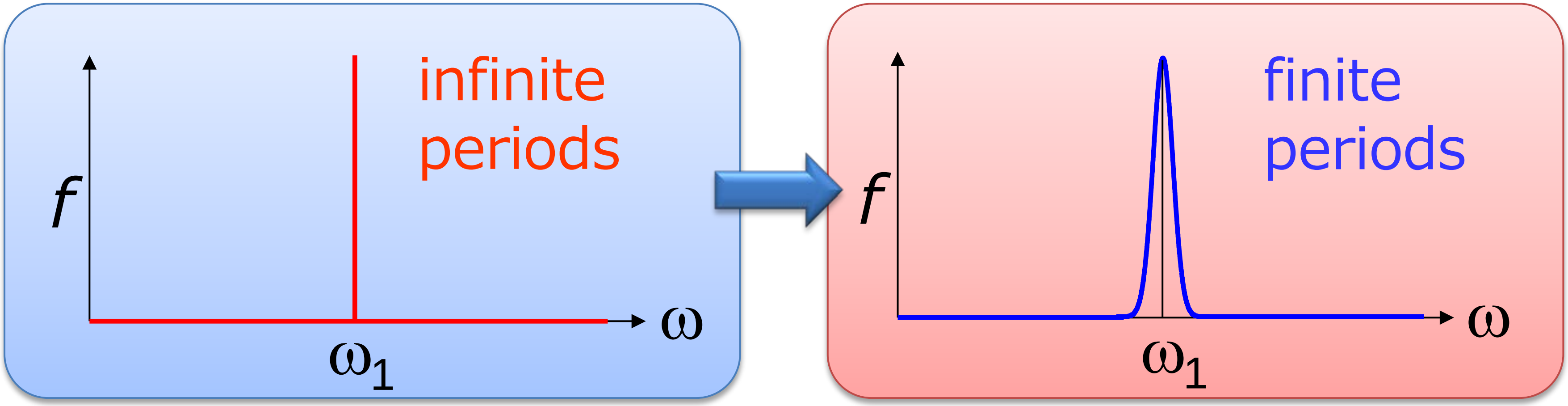
$$\begin{aligned} \lambda_1 &= \lambda_u(1-\bar{\beta}_z\cos\theta) \\ &= \frac{\lambda_u}{2\gamma^2}(1+\gamma^2\theta^2+K^2/2) \\ \omega_1 &= 2\pi c/\lambda_1 \end{aligned}$$

UR with Infinite Periods

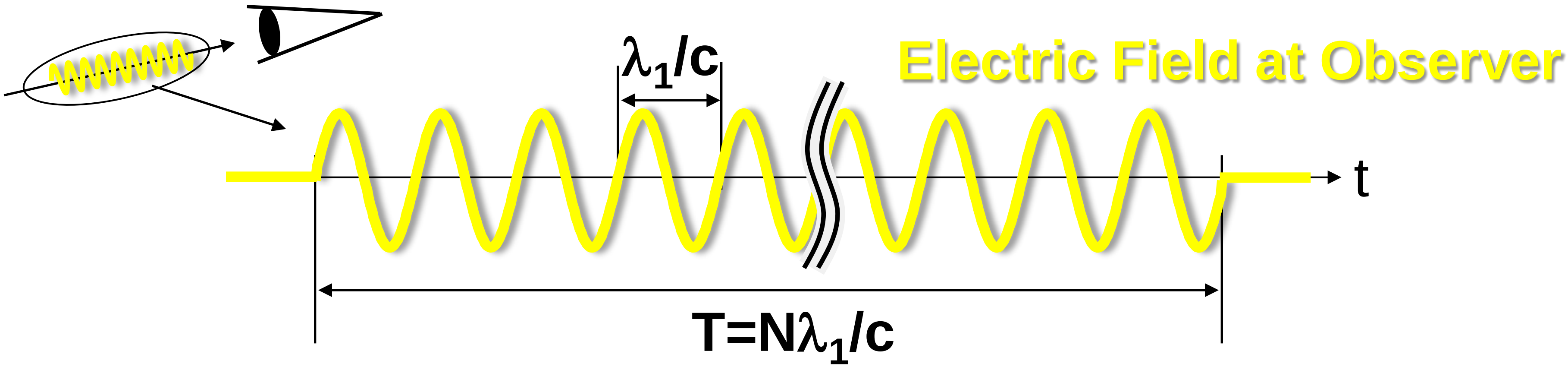
- If the undulator length is infinite, the pulse duration is infinitely long, and thus the radiation is completely monochromatic with line spectrum.

$$f(\theta,\omega)=\delta(\omega-\omega_1)=\delta\left(\omega-\frac{4\pi c\gamma^2/\lambda_u}{1+K^2/2+\gamma^2\theta^2}\right)$$

- In practice, the undulator length is finite, so the line spectrum is broadened.



Effects due to Finite Periods



$$E(t) = \begin{cases} E_0 \sin \omega_1 t & ; -T/2 \le t \le T/2 \\ 0 & ; t < -T/2,\ \ T/2 < t \end{cases} \quad , \quad \omega_1 = 2\pi c/\lambda_1$$

Fourier Transform

$$f(\theta,\omega) \propto |\tilde{E}(\omega)|^2 \propto \mathrm{sinc}^2 \left[\pi N \frac{\omega - \omega_1(\theta)}{\omega_1(\theta)} \right]$$

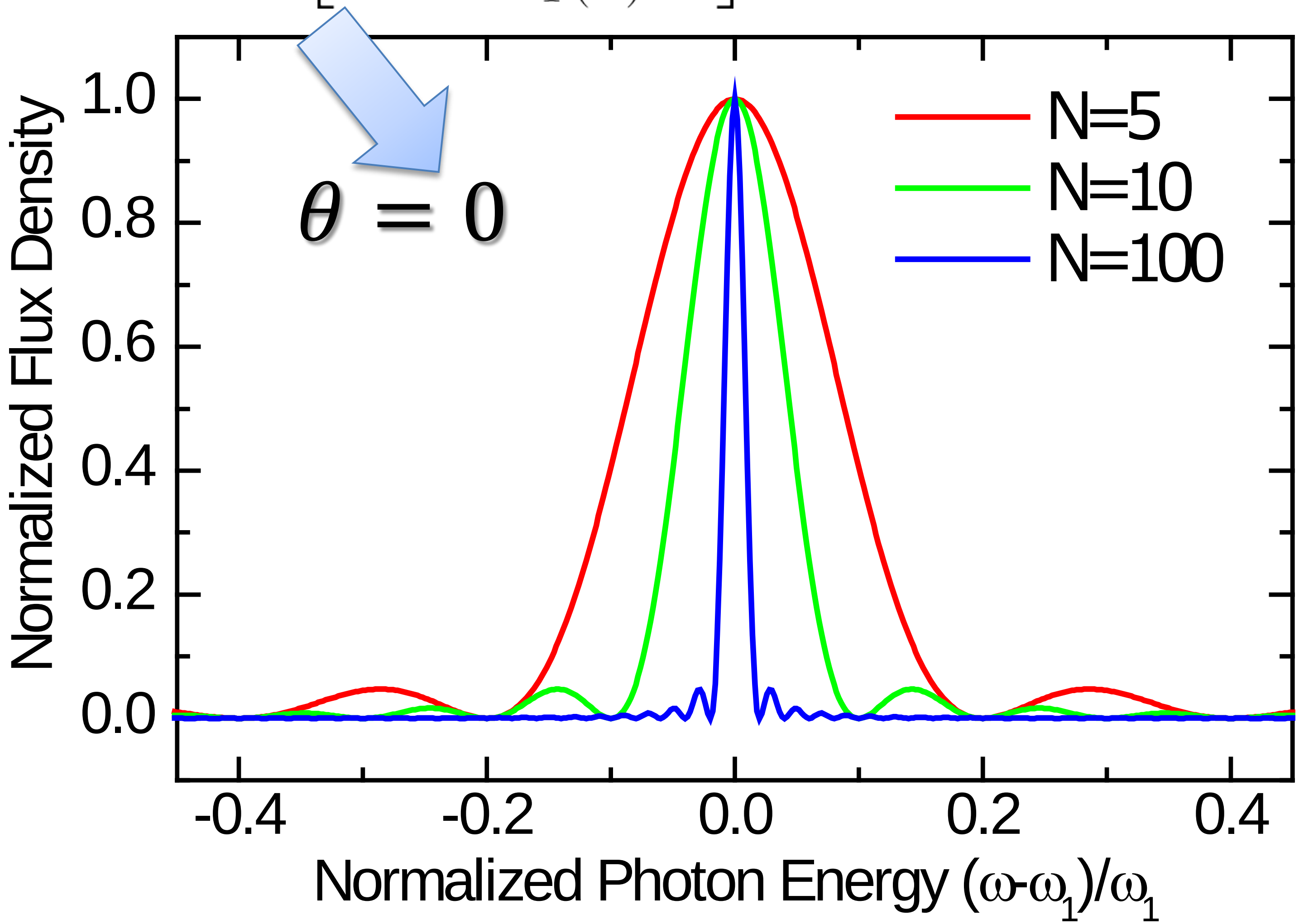
Square of "sinc" function dominates UR

Brief Note on UR Formulae

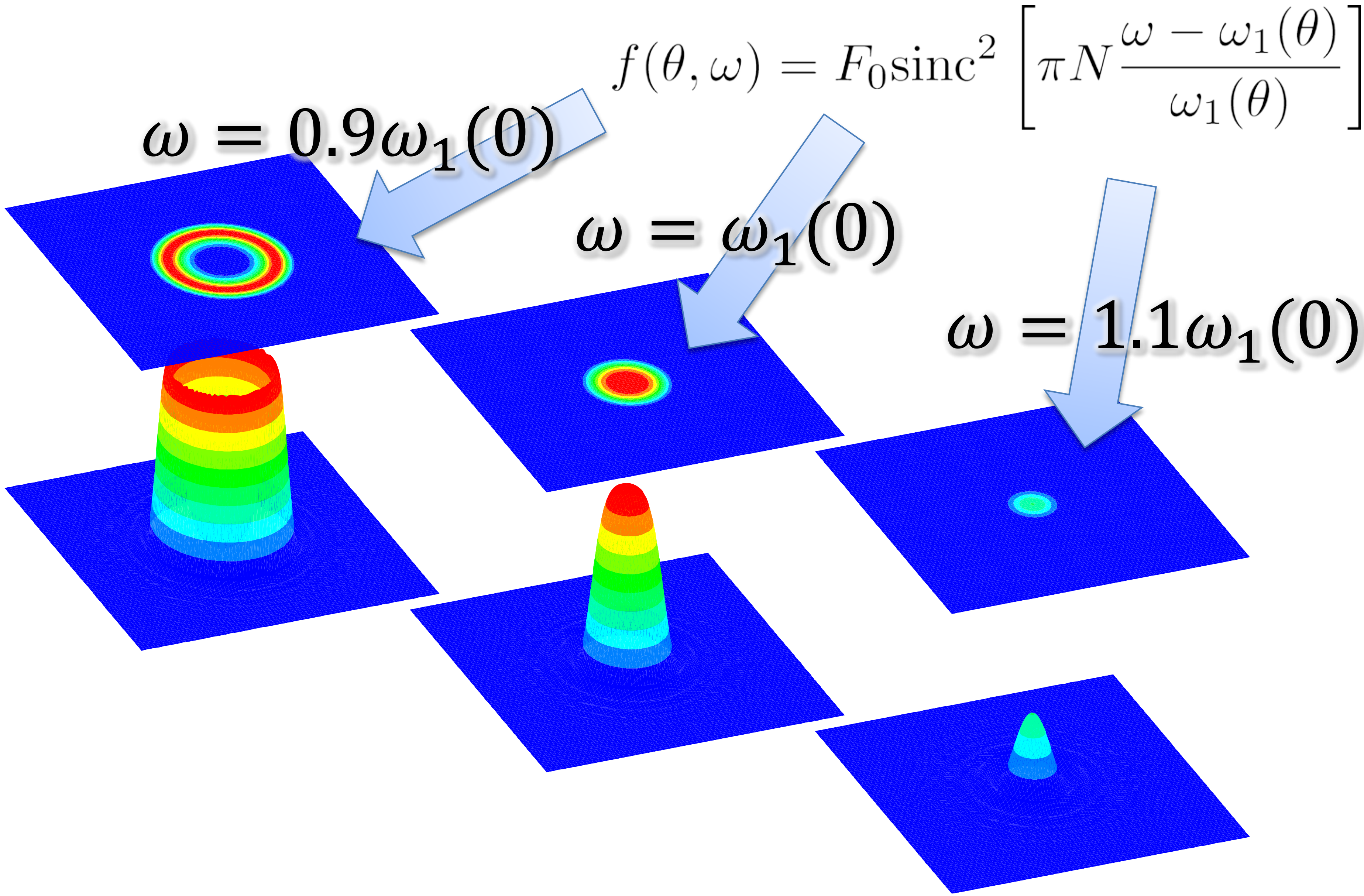
- In the previous derivations of UR spectral function, no knowledge on electrodynamics is required.
- In practice, E_0 is a complicated function of θ and K, and needs to be calculated by Fourier transforming the electric field derived from the Lienard-Wiecherd potential.
- However, the simple derivation gives us a clear understanding on UR properties.

Energy Spectrum of UR

$$f(\theta, \omega) = F_0 \mathrm{sinc}^2 \left[\pi N \frac{\omega - \omega_1(\theta)}{\omega_1(\theta)} \right]$$



Angular Profile of UR

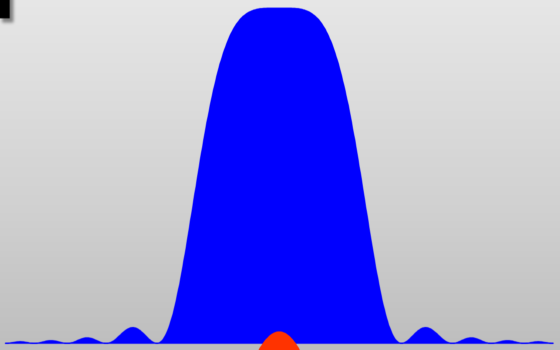


Angular Divergence of UR

UR is not a Gaussian Beam

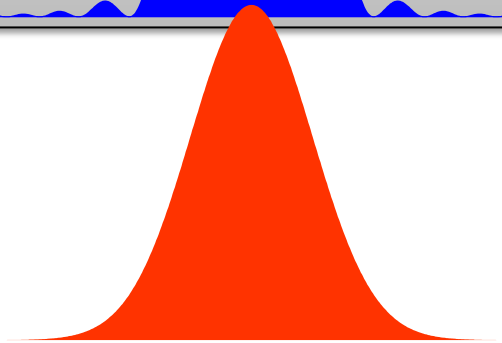
Angular Profile at $\omega=\omega_1(0)$

$$f(\theta,\omega) = F_0 \mathrm{sinc}^2\left[\frac{\pi N(\gamma\theta)^2}{1+K^2/2}\right]$$



Gaussian Profile with $\sigma_{r'}$

$$f_G(\theta) = F_0 \exp\left(-\frac{\theta}{2\sigma_r'^2}\right)$$



$$\int f(\theta,\omega)d\boldsymbol{\theta} = \int f_G(\theta)d\boldsymbol{\theta}$$

$$\sigma_{r'} = \sqrt{\frac{1+K^2/2}{4N\gamma^2}} = \sqrt{\frac{\lambda_1}{2L}}$$

"Natural" Angular Divergence of UR ($L = N\lambda_u$)

Source Size of UR

$$\sigma_{r'} = \sqrt{\frac{1+K^2/2}{4N\gamma^2}} = \sqrt{\frac{\lambda_1}{2L}}$$

"Natural" Angular Divergence of UR ($L = N\lambda_u$)

- ✓ Unlike WR, UR is spatially coherent, i.e., diffraction limited.
- ✓ This means that $\sigma_r \sigma_{r'} = \lambda/4\pi$

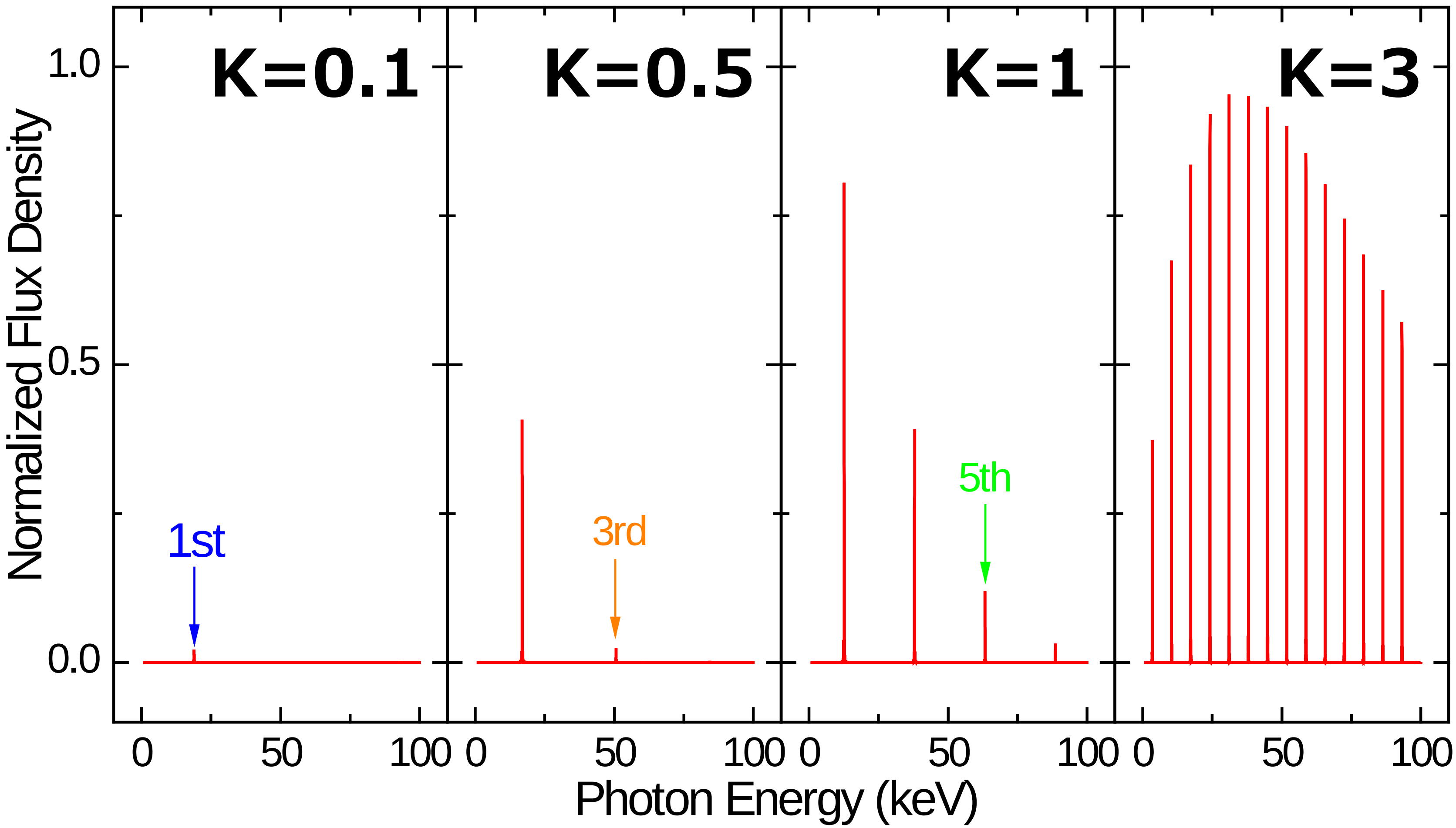
$$\sigma_r = \frac{\lambda_1}{4\pi\sigma_{r'}} = \frac{\sqrt{2L\lambda_1}}{4\pi}$$

"Natural" Source size of UR

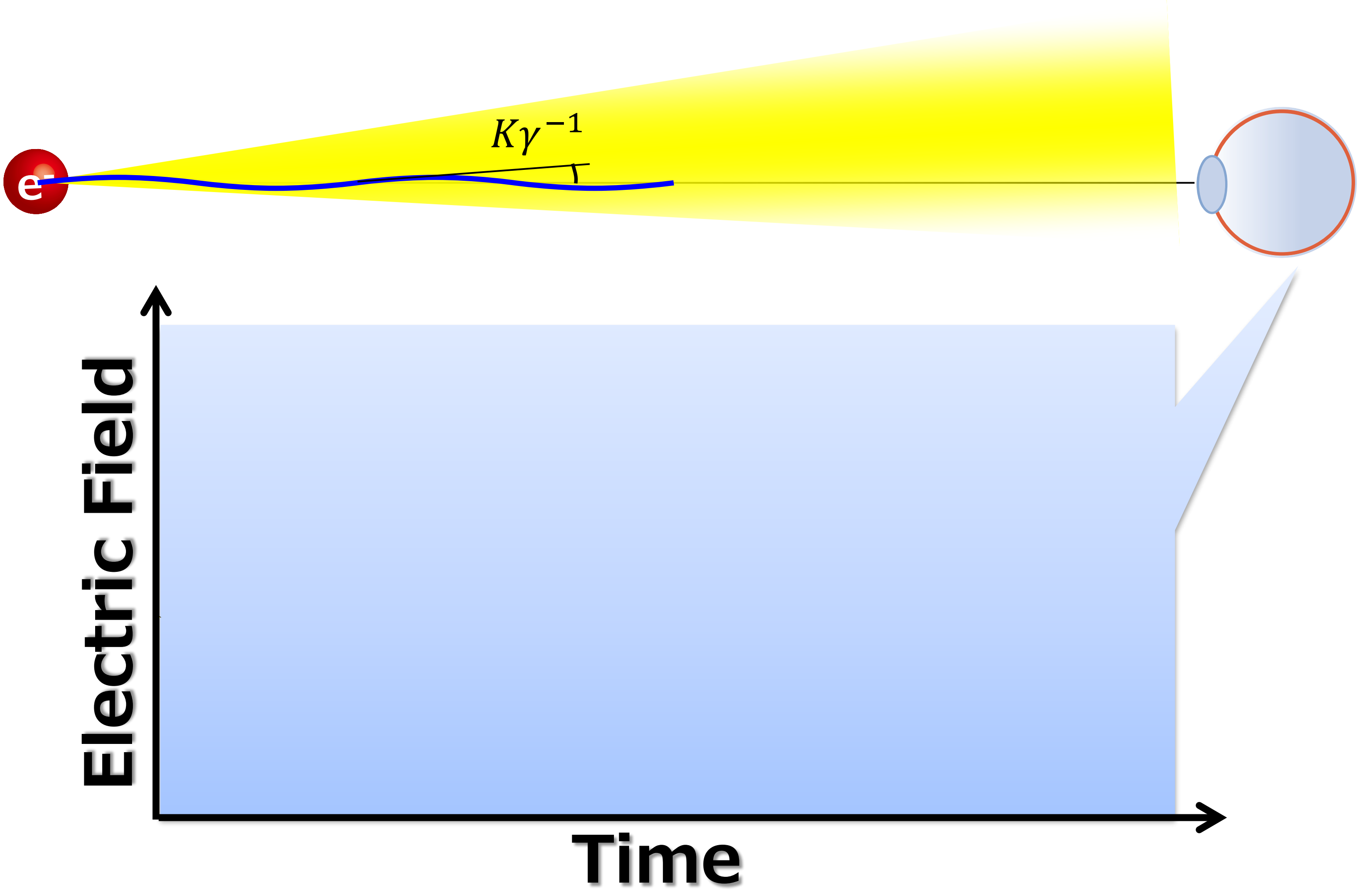
Longer device results in smaller angular divergence & larger source size, but the emittance does not change.

Higher Harmonics

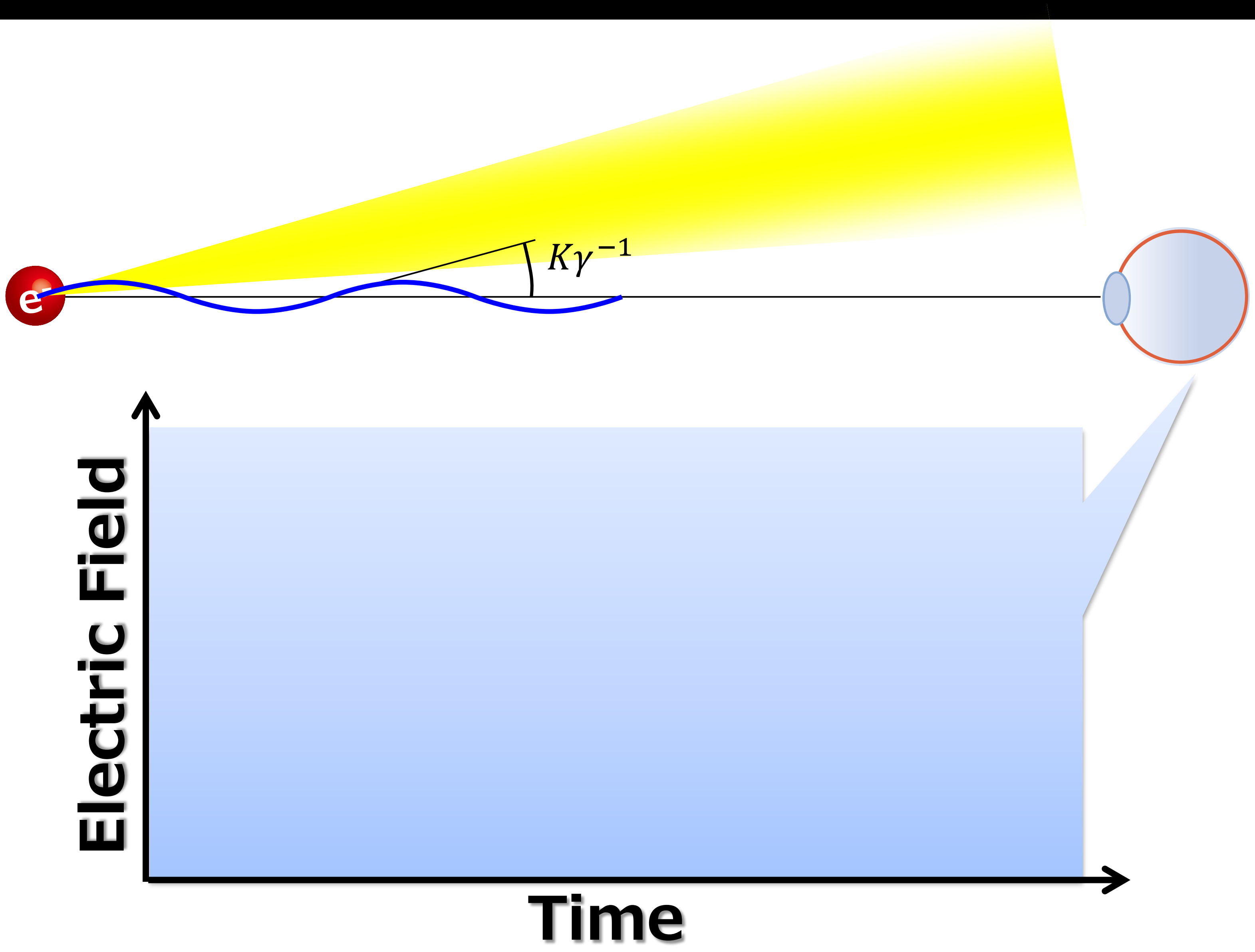
Photons with at $n\omega_1$ are observed as well as at ω_1, where n is an integer.



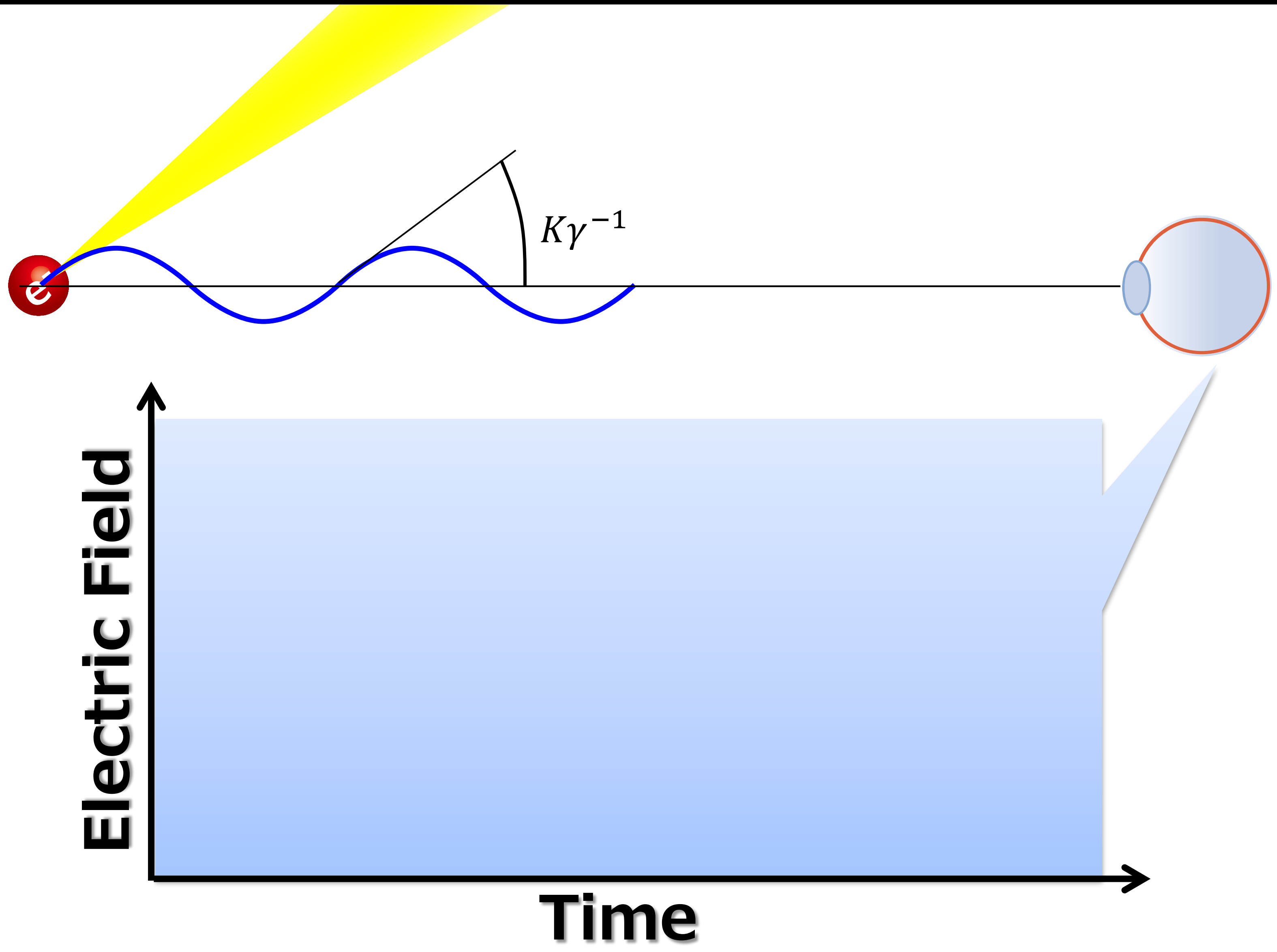
K << 1 Case



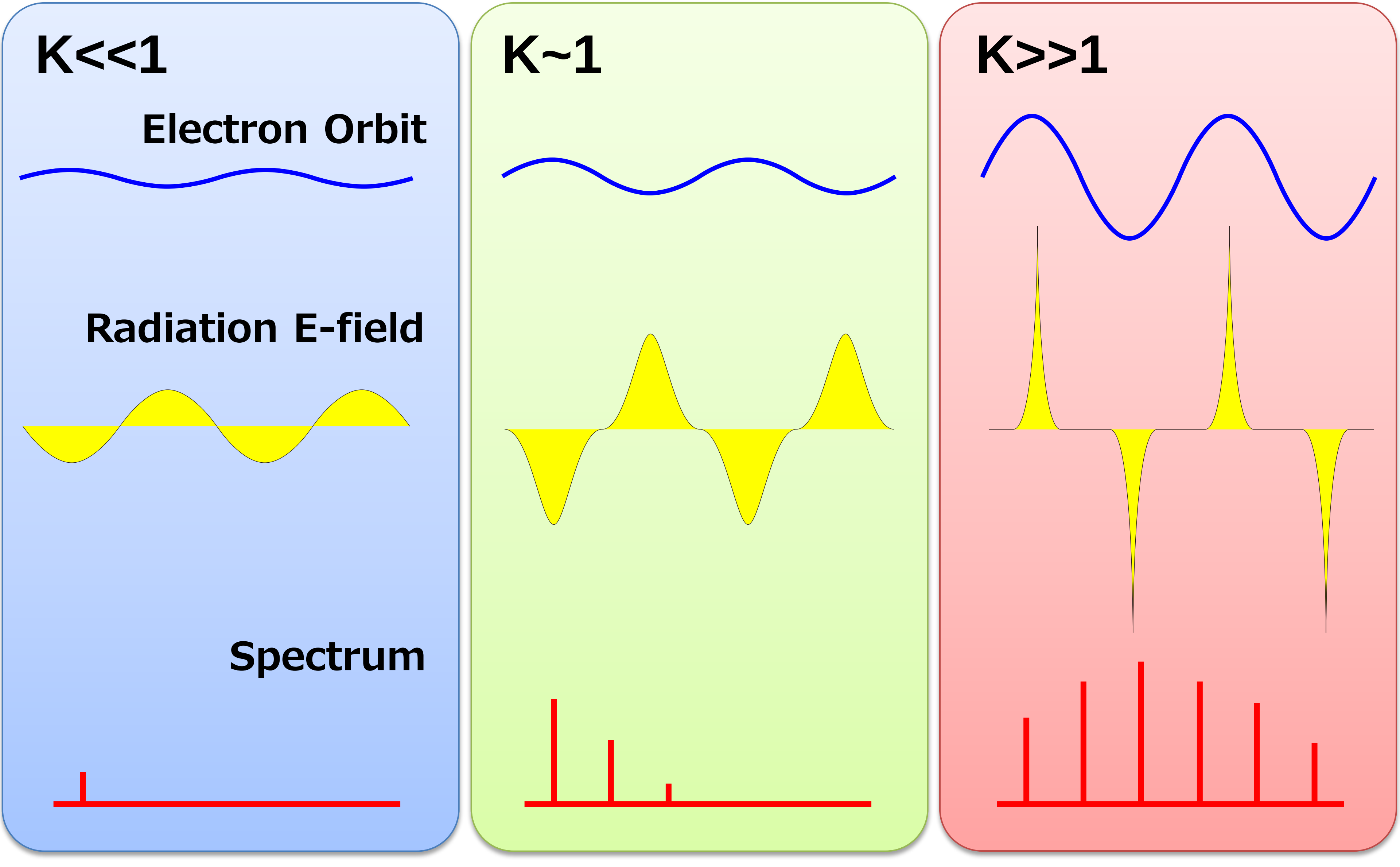
K ~ 1 Case



K >> 1 Case



Mechanisms of Higher Harmonics

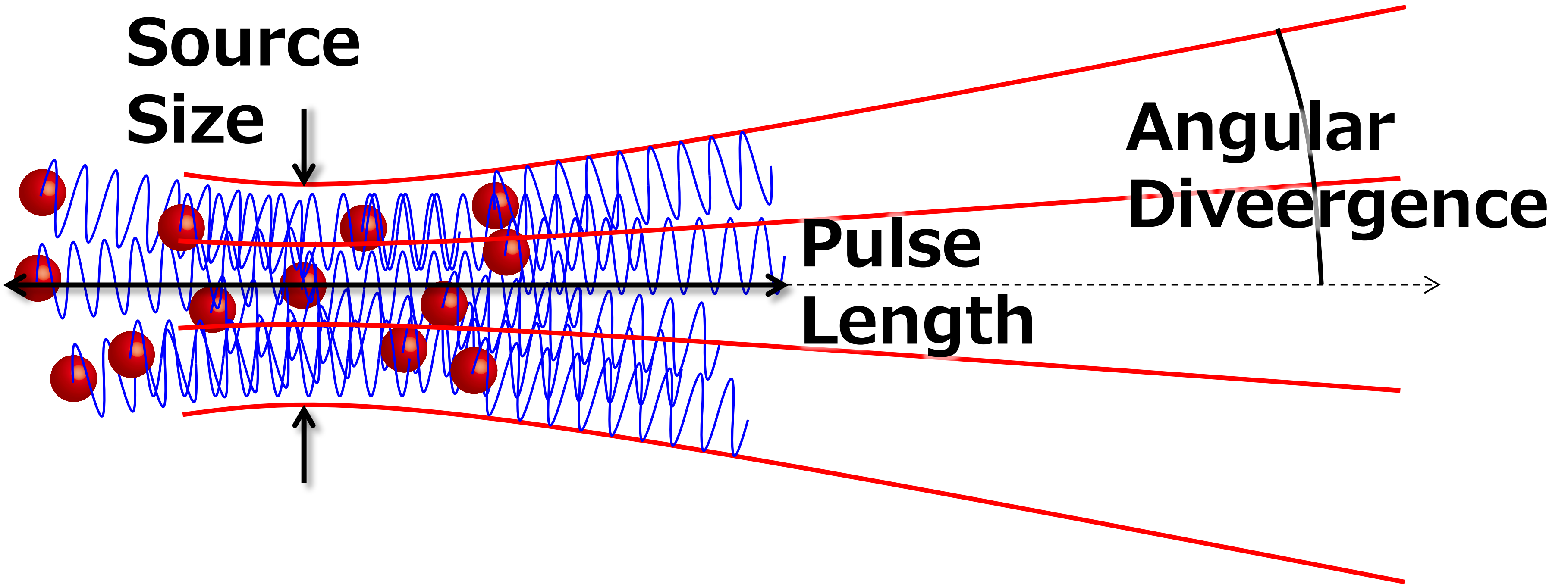


Outline

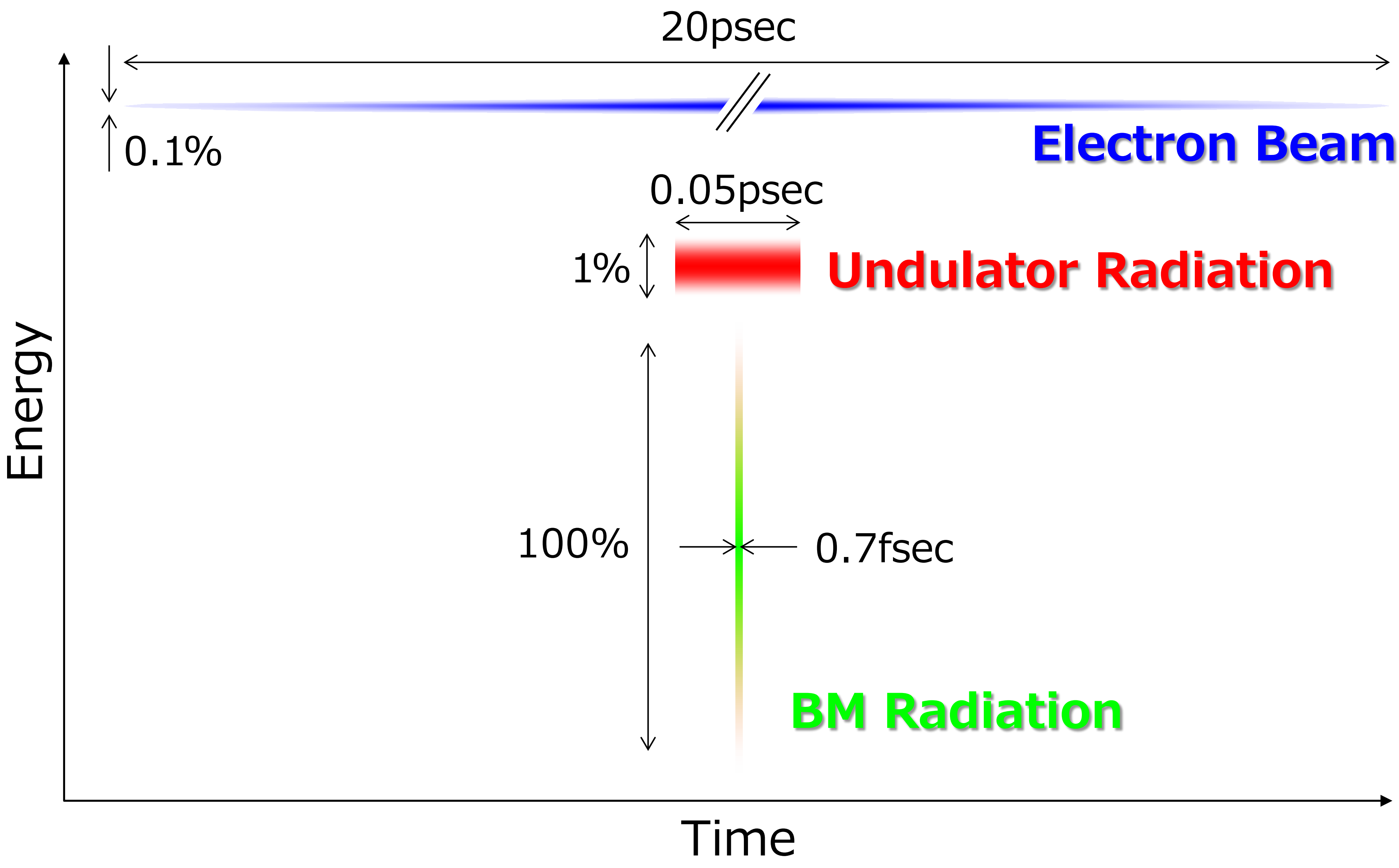
- Introduction
- Fundamentals of Light and SR
- Overview of SR Light Source
- Characteristics of SR (1)
- Characteristics of SR (2)
- **Practical Knowledge on SR**

Effective Properties of SR

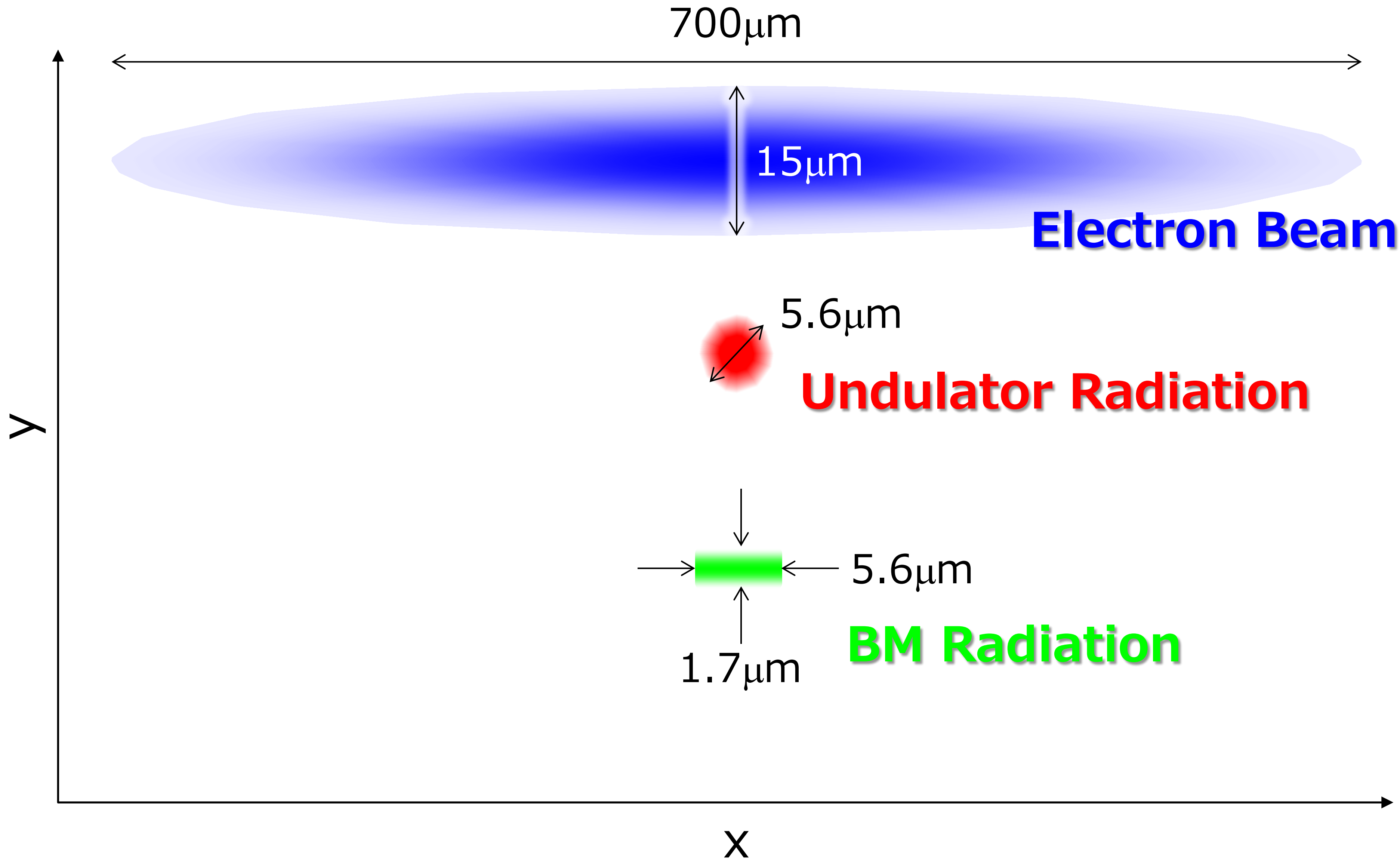
- Properties of SR emitted from an e^- beam are different from those from a single e^-.
- They are referred to as "effective" properties of SR, as opposed to "natural" properties.



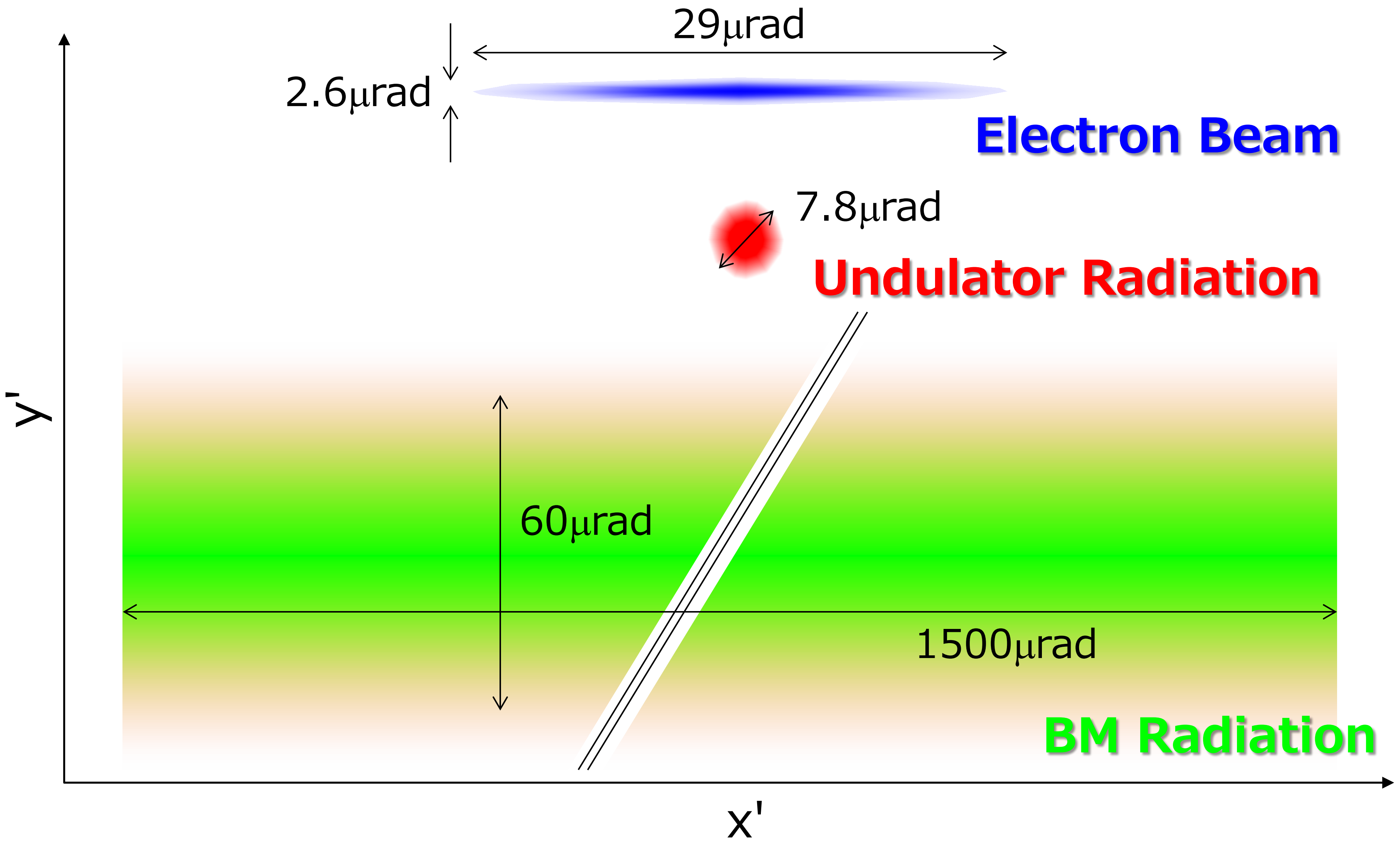
Example in SPring-8: E-t Phase Space



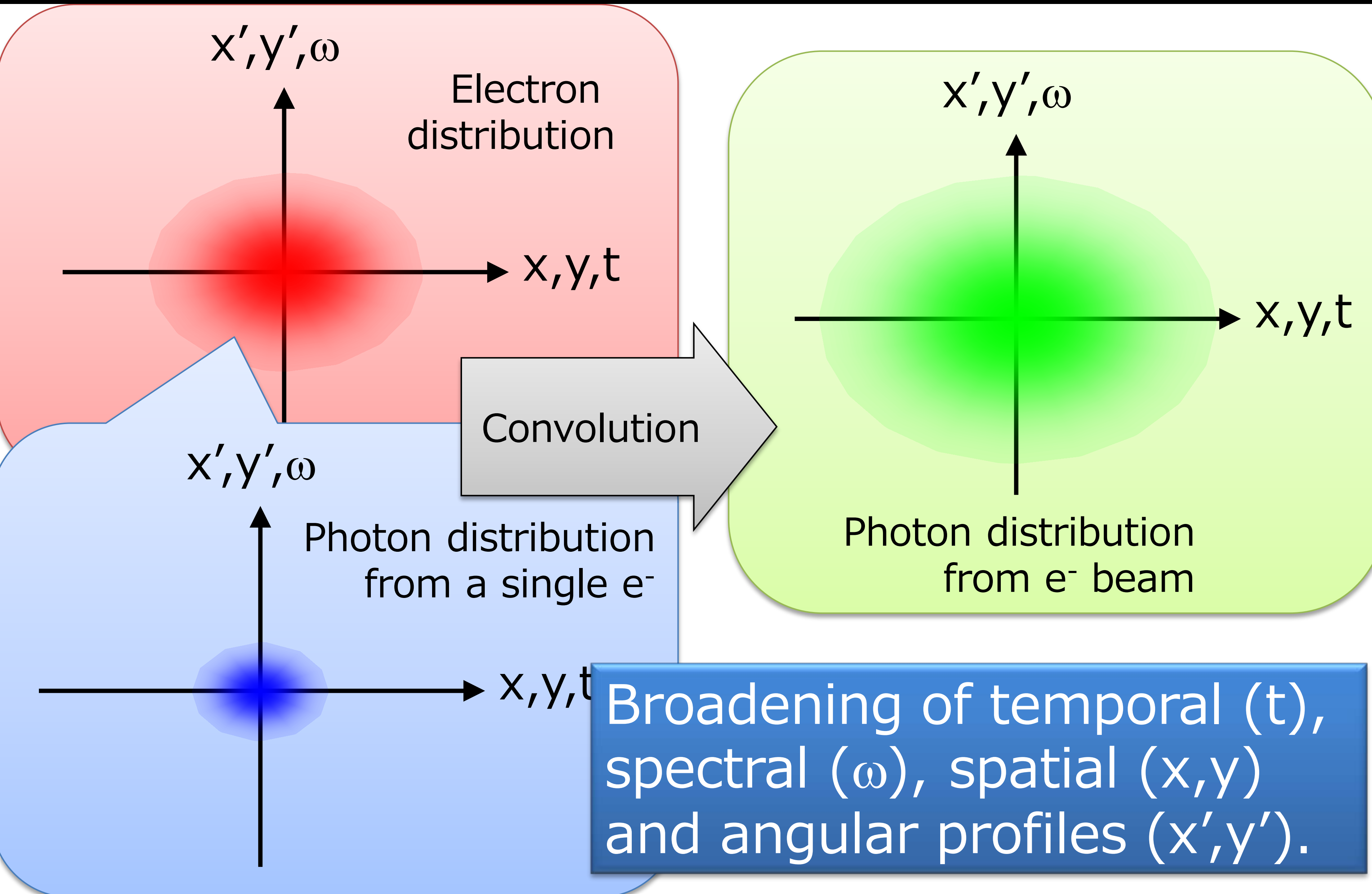
Example in SPring-8: (x,y) Space



Example in SPring-8: (x′,y′) Space

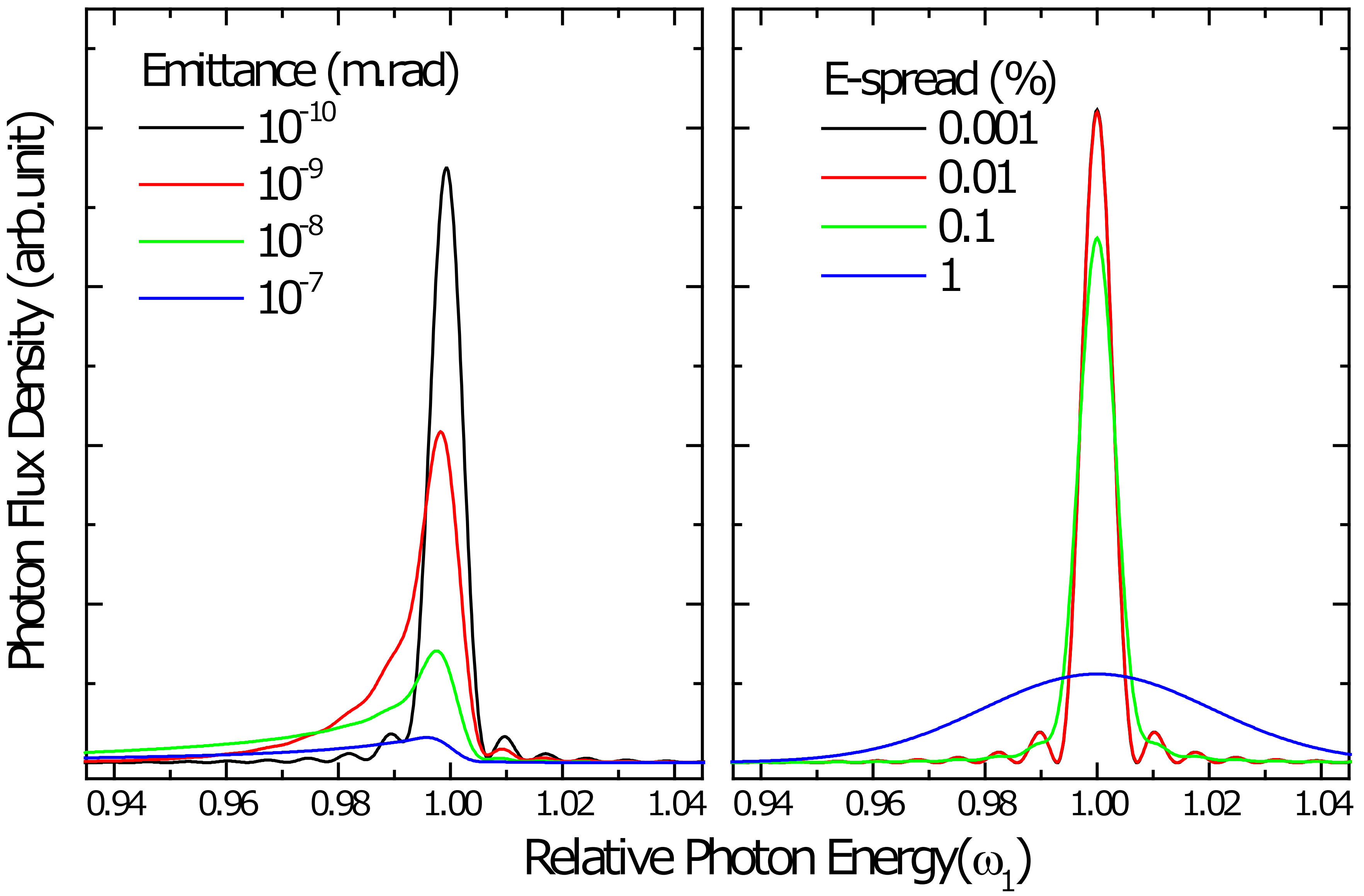


Convolution Between e- and Photon

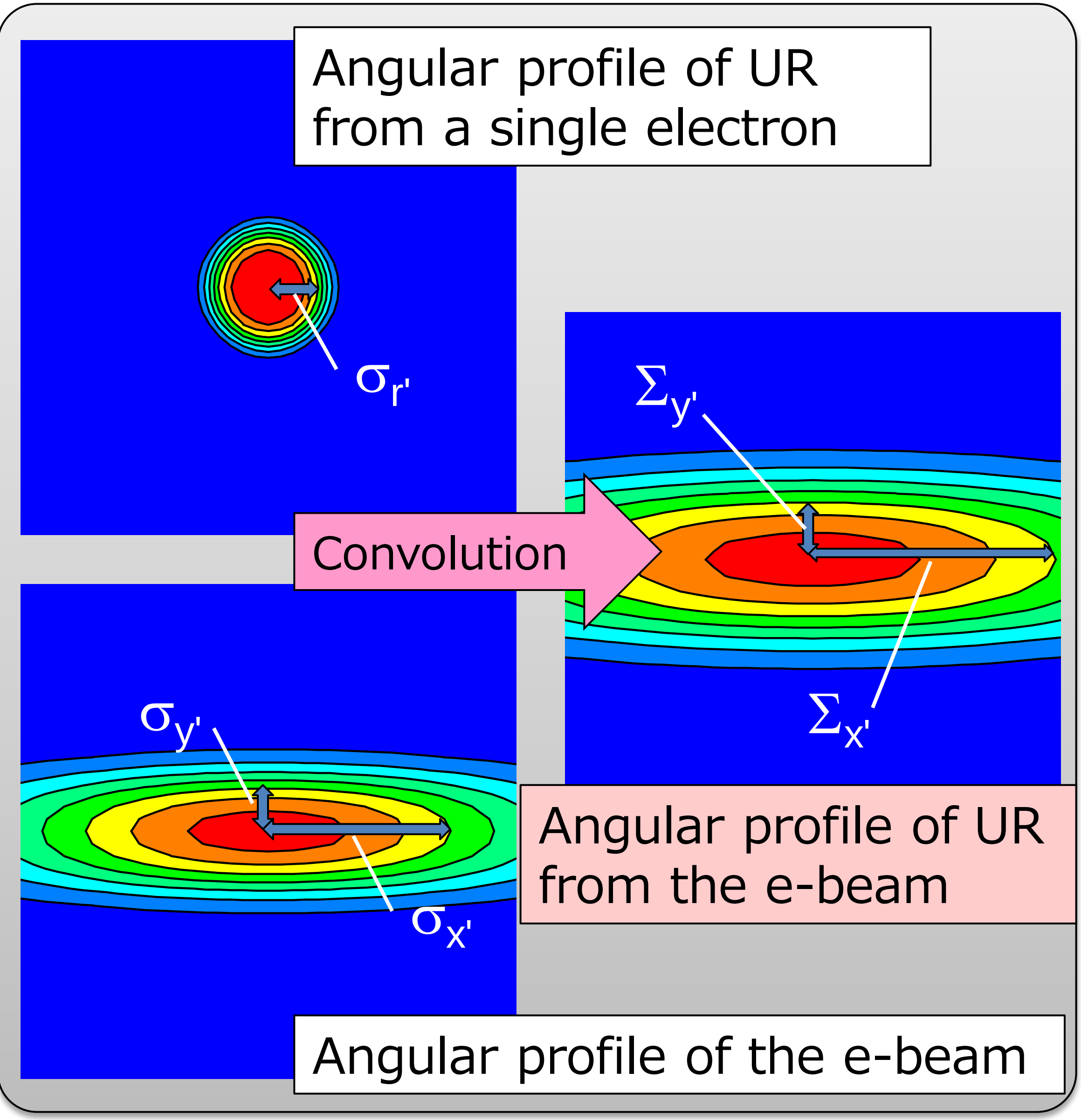


Broadening of temporal (t), spectral (ω), spatial (x,y) and angular profiles (x′,y′).

Spectral Profile (UR)



Angular & Spatial Profile (UR)



✓Gauss approximation
✓Convolution theorem

$$\Sigma_{x',y'} = \sqrt{\sigma_{r'}^2 + \sigma_{x',y'}^2}$$

Effective Angular Div.

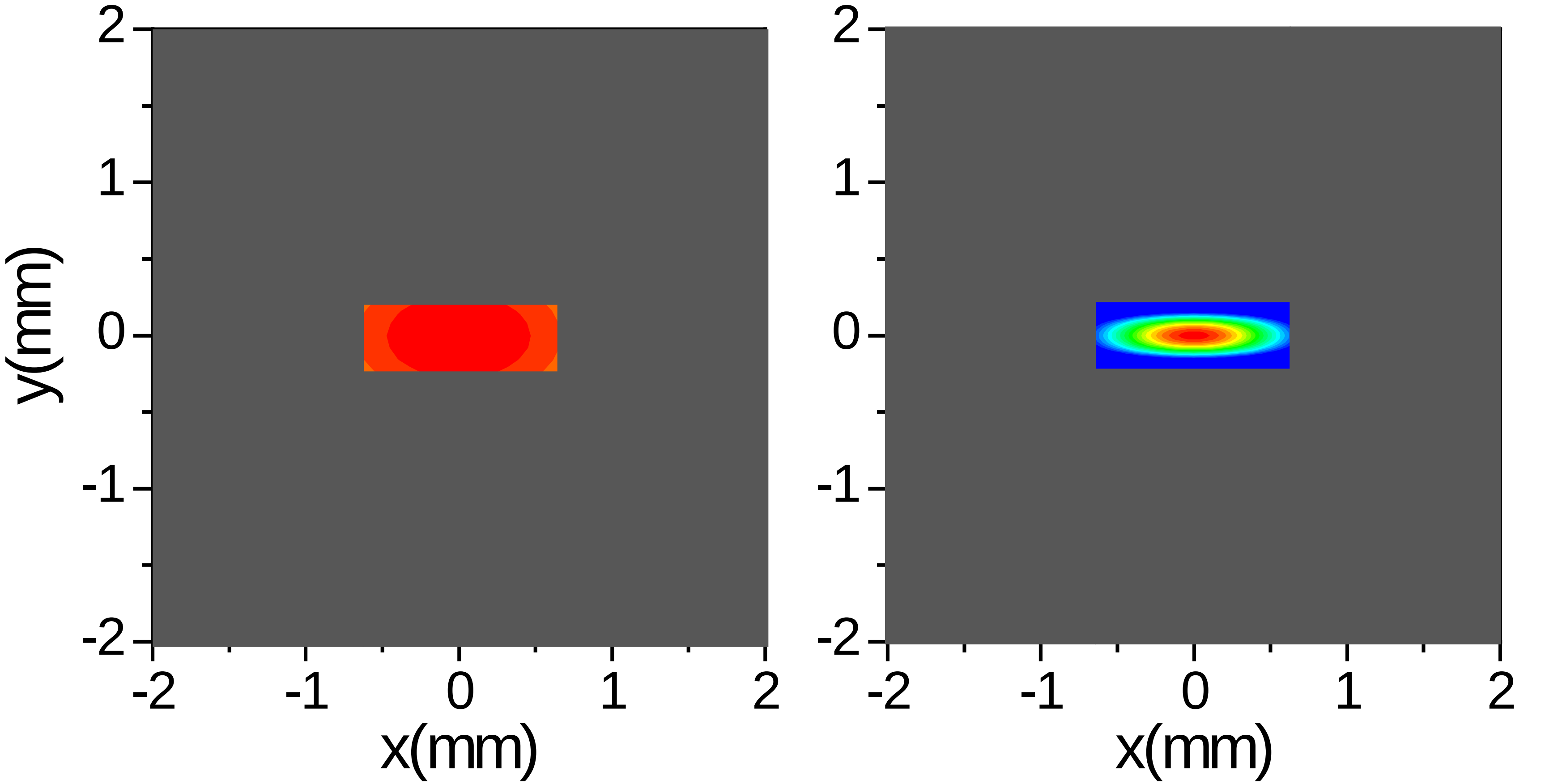
$$\Sigma_{x,y} = \sqrt{\sigma_r^2 + \sigma_{x,y}^2}$$

Effective Source Size

Heat Load on Optical Elements

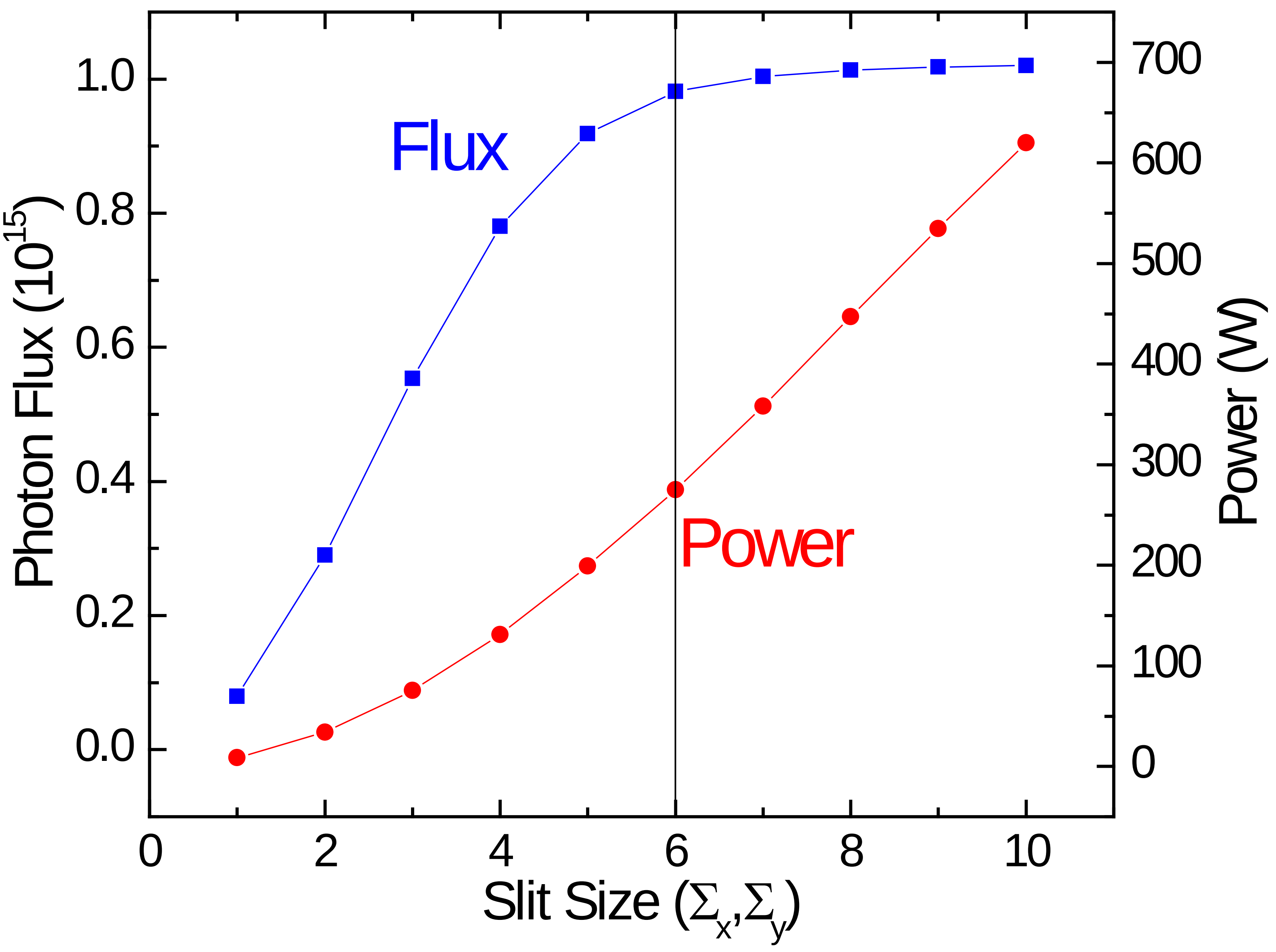
- SR is usually processed by several optical elements before irradiation to the sample, such as the focusing mirror, monochromator.
- These elements can be easily damaged by the heat load of SR.
- In the case of UR, the heat load can be reduced by taking advantage of the difference in the angular profile of the photon flux and radiation power.

Spatial Profile of Power and Flux (UR)



The power profile is much broader than the flux.
Extraction of SR with an appropriate slit is thus effective.

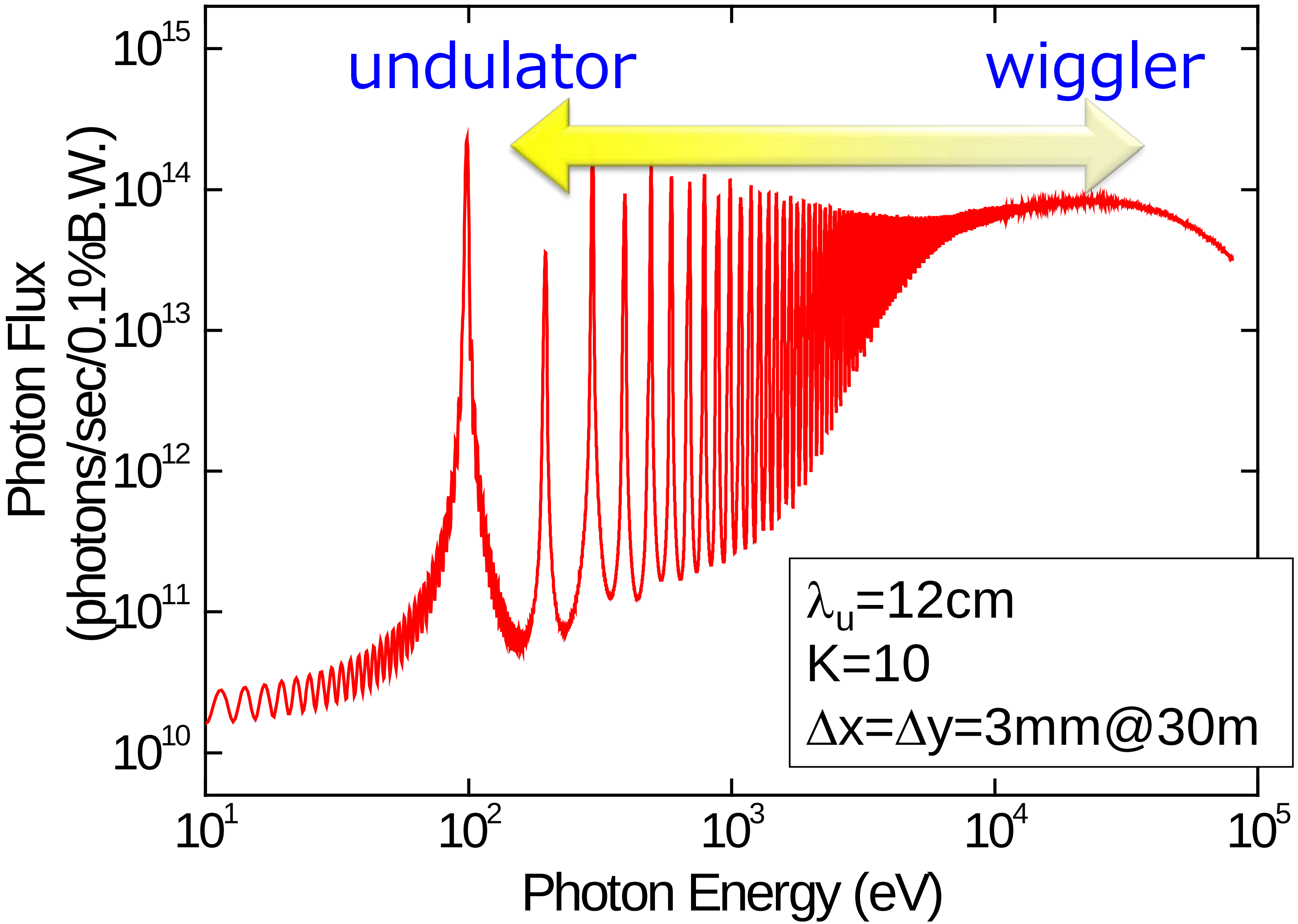
What’s the Optimum Aperture Size?



Wiggler? Undulator? (1)

- Wigglers are identical to undulator from the point of view of magnetic circuit.
- It is generally said that the K value distinguishes between the two, however, this is not exactly correct.
- What we should take care is the photon energy region of interest.

Wiggler? Undulator? (2)

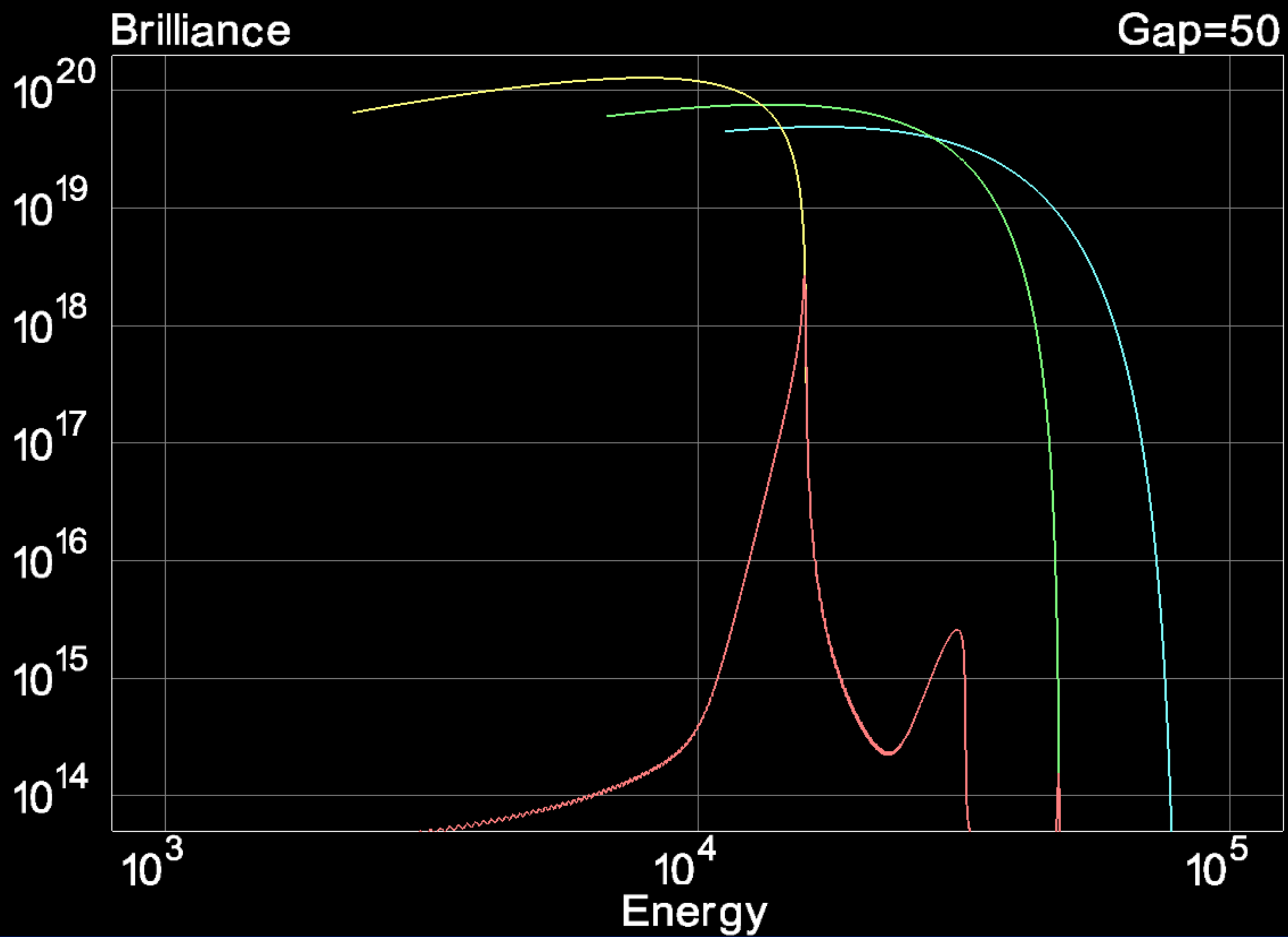


Undulator Radiation Gallery

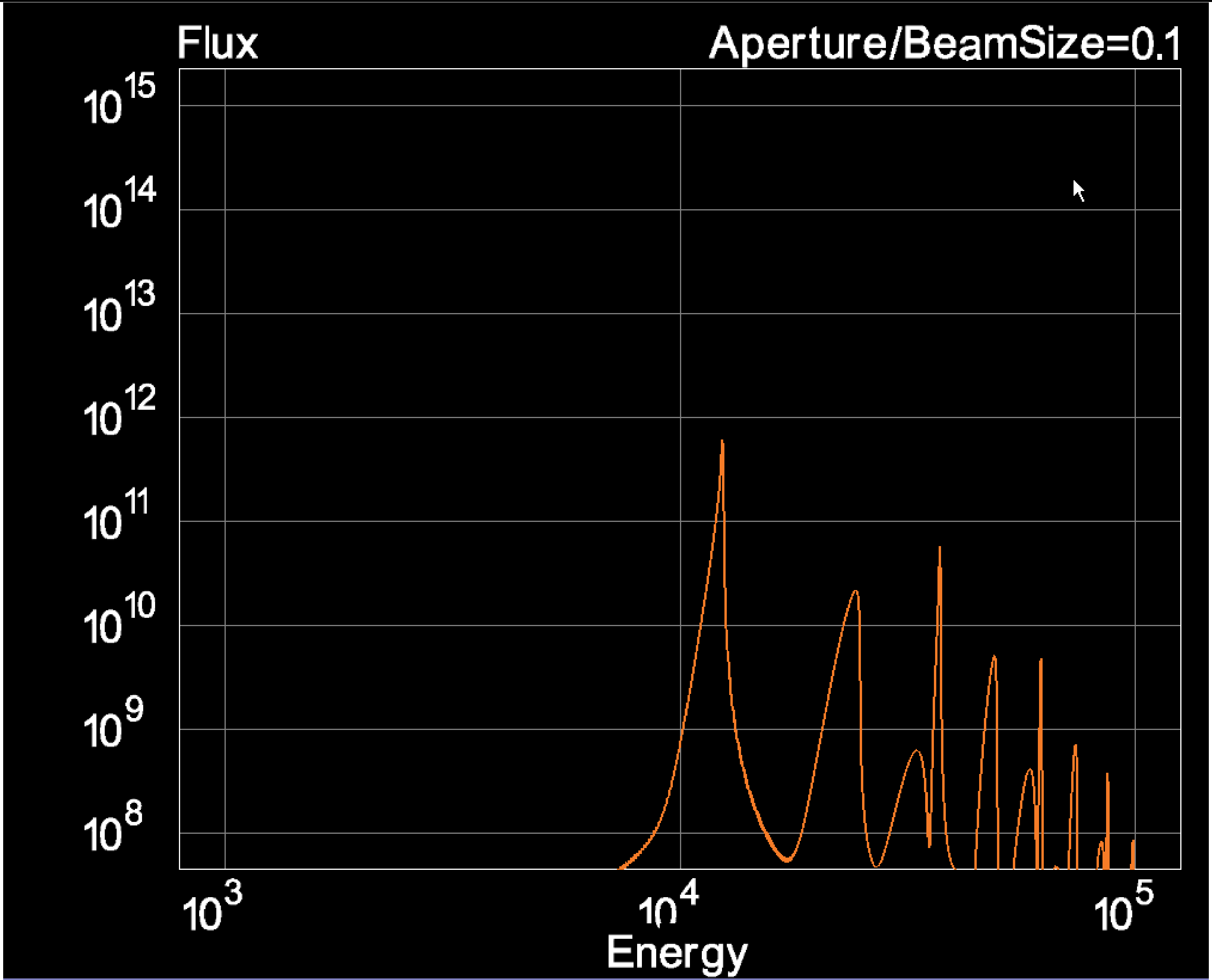
- For quantitative evaluation of SR, a computer code “SPECTRA” is available.
- SPECTRA also offers a function to “visualize” the computation results for further understanding of SR.
  - brilliance curve & spectrum
  - on- and off-peak angular profiles of flux
  - on- and off-axis spectra
  - effects of opening the slit aperture
  - undulator-to-wiggler transition

Brilliance Curve & Spectrum

Spectrum –, Peak Brilliance 1st – 3rd – 5th –

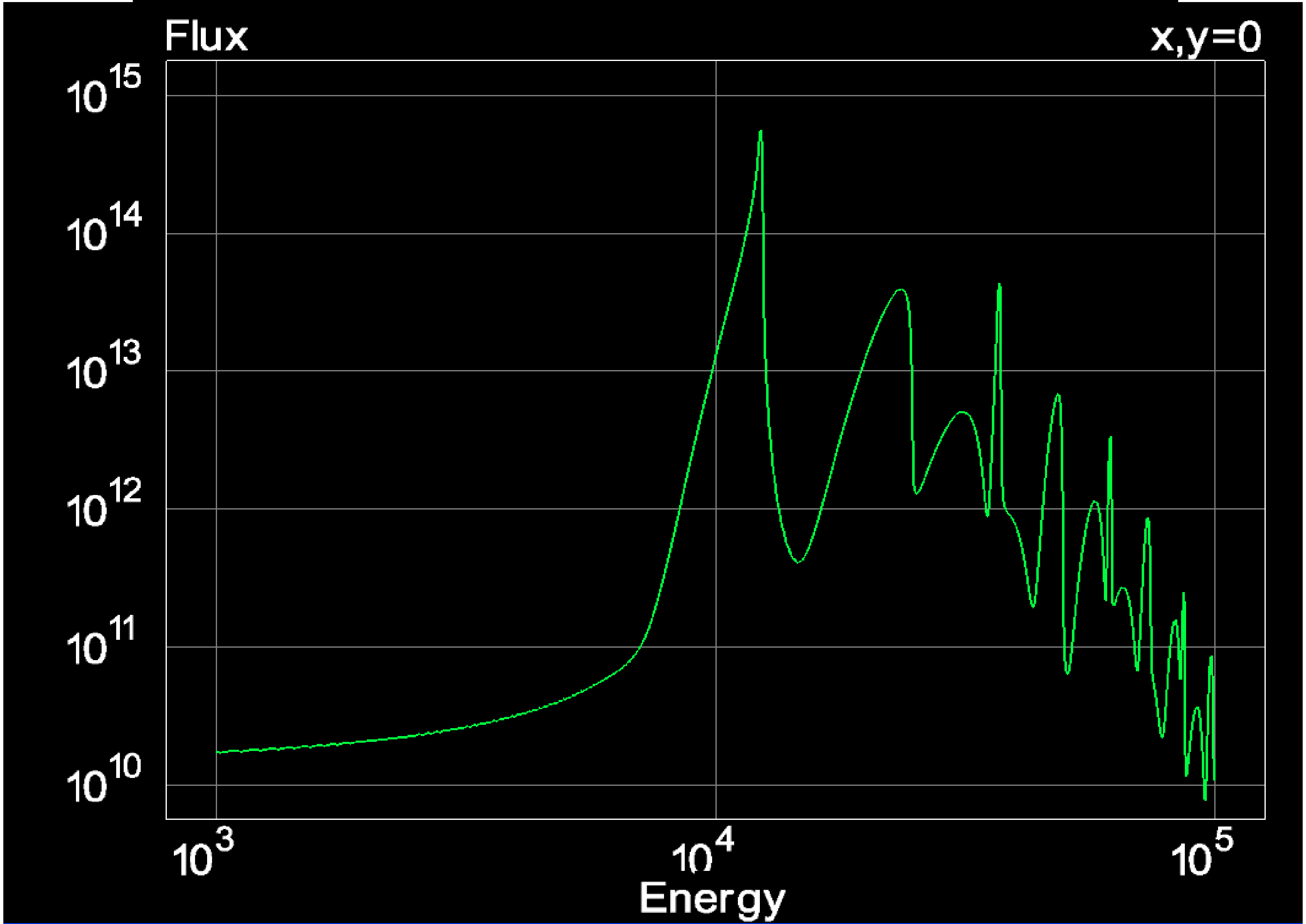


Opening the Slit Aperture



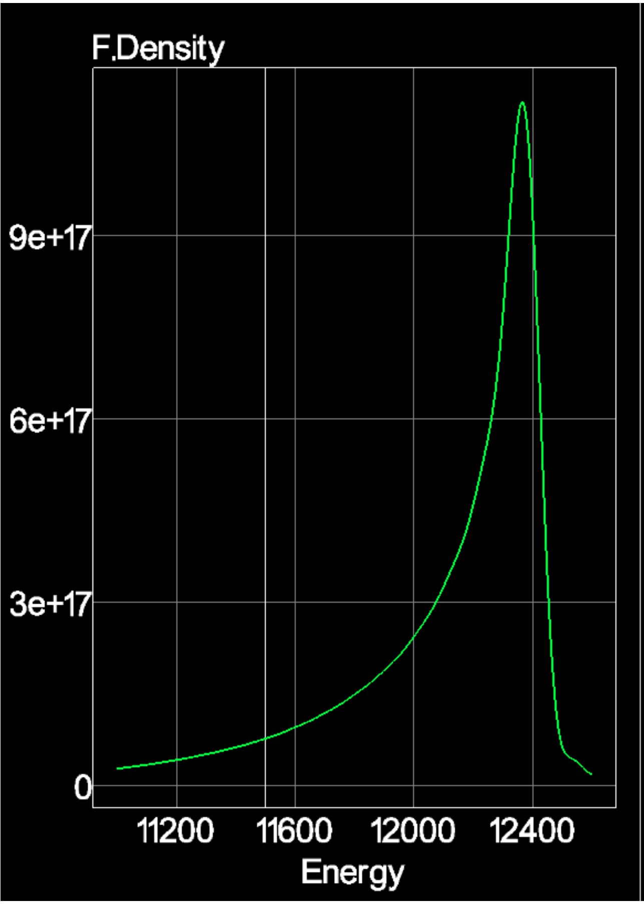
Off-Axis Spectrum

Moving the slit along – x, – y

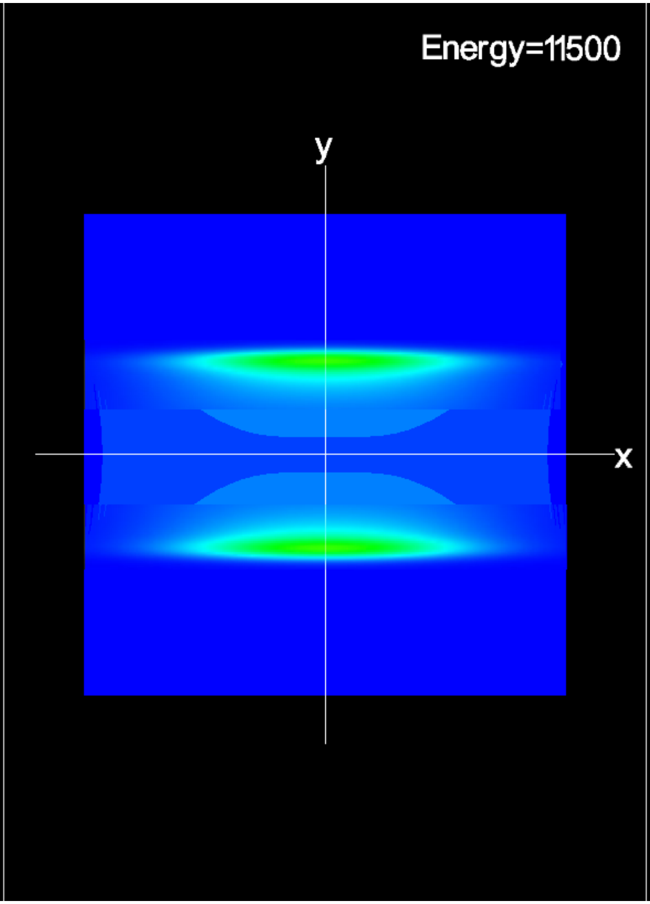


Flux Angular Profile

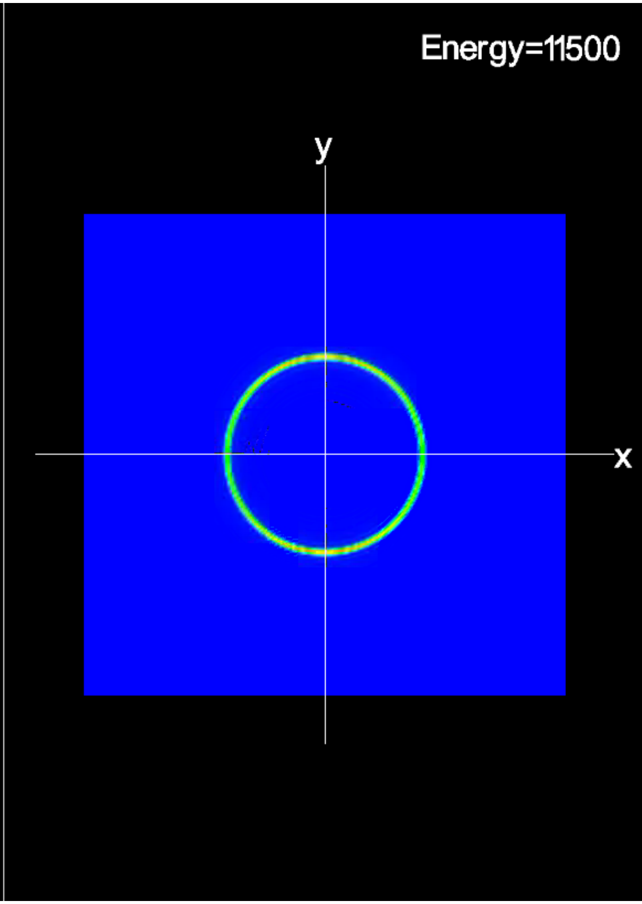
On-Axis Spectrum



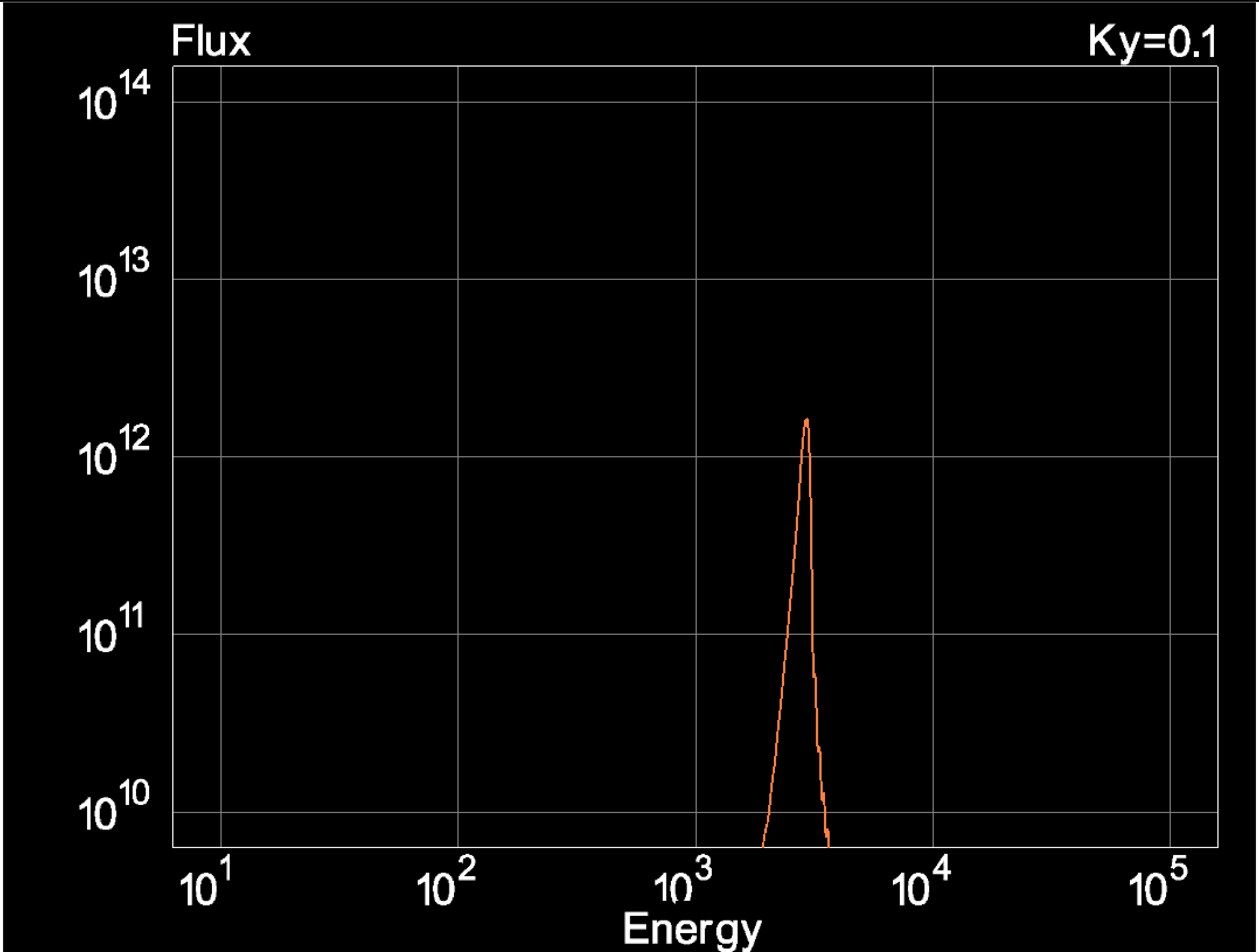
Angular Profile (Finite Emittance)



Angular Profile (Zero Emittance)



Undulator-to-Wiggler Transition



Angular Divergence & Source Size